Adjustment and social choice

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Some behaviour observed on markets seem quite different form the prediction of the economic theory such as a ”smooth balance” between supply and demand through price adjustment. (basic hypothesis of GET: both supply and demand have finite derivative with respect to prices). The purpose of the present talk is to:

- Social choice may result in sharp transitions in the demand curve: in the absence of adjustment by economic agents, one may predict a market of either hits or flops as observed in the gadget or media industry.
- In the presence of re-adjustment by buyers and sellers, either through price or quality of products, a *self-organised critical* regime with large non gaussian fluctuation of purchase is predicted (c.f. histograms of returns in financial markets).

"Static" models of social choice

Basics: in situations with incomplete information, economic agents are influenced by their neighbours' choice. This is an instance of "bounded rationality", a set of hypotheses distinct from the "standard" full rationality hypothesis of economic theory.

2 possible implementations inspired from physics:

- Get the info, e.g. product quality form any one of your neighbourswho purchased (formal equiv. to percolation).
- Gather info, e.g. "how many purchased", among all your neighboursand balance it against some private info (formal equiv. to seeds and crystal growth).

Both implementation give abrupt transition in purchase/quality (or price) relation.

Further assumptions of the models presented (not necessary conditions):

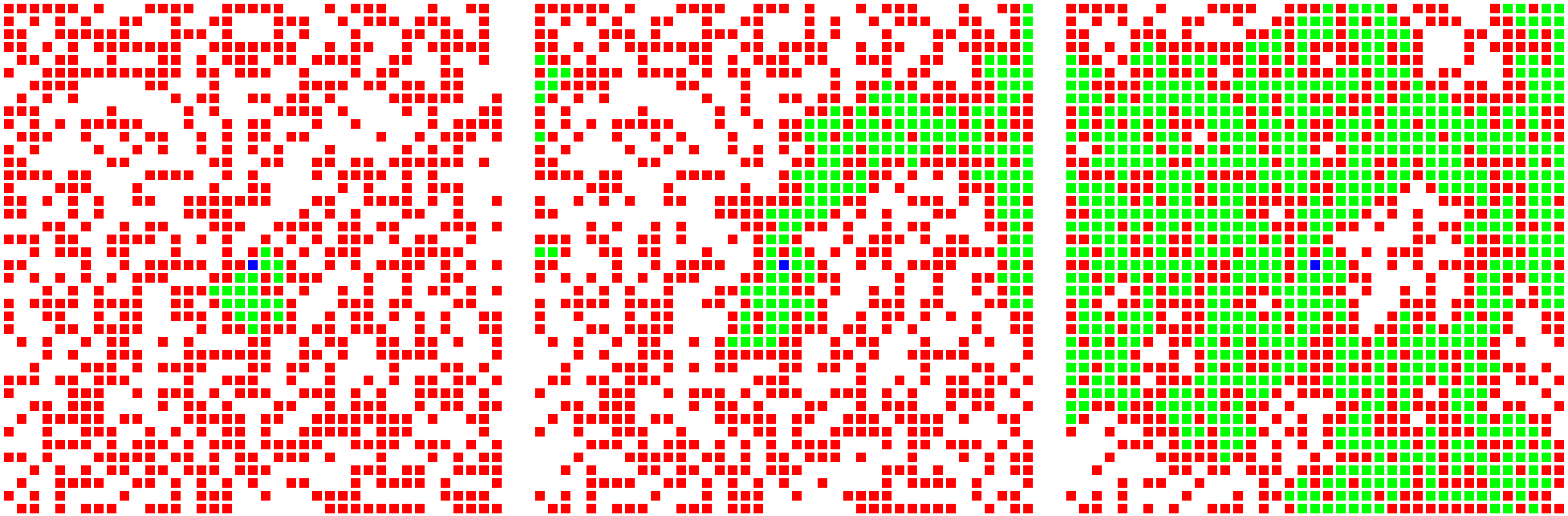
- heterogeneity of buyers
- square lattice as interaction graphs

The percolation model

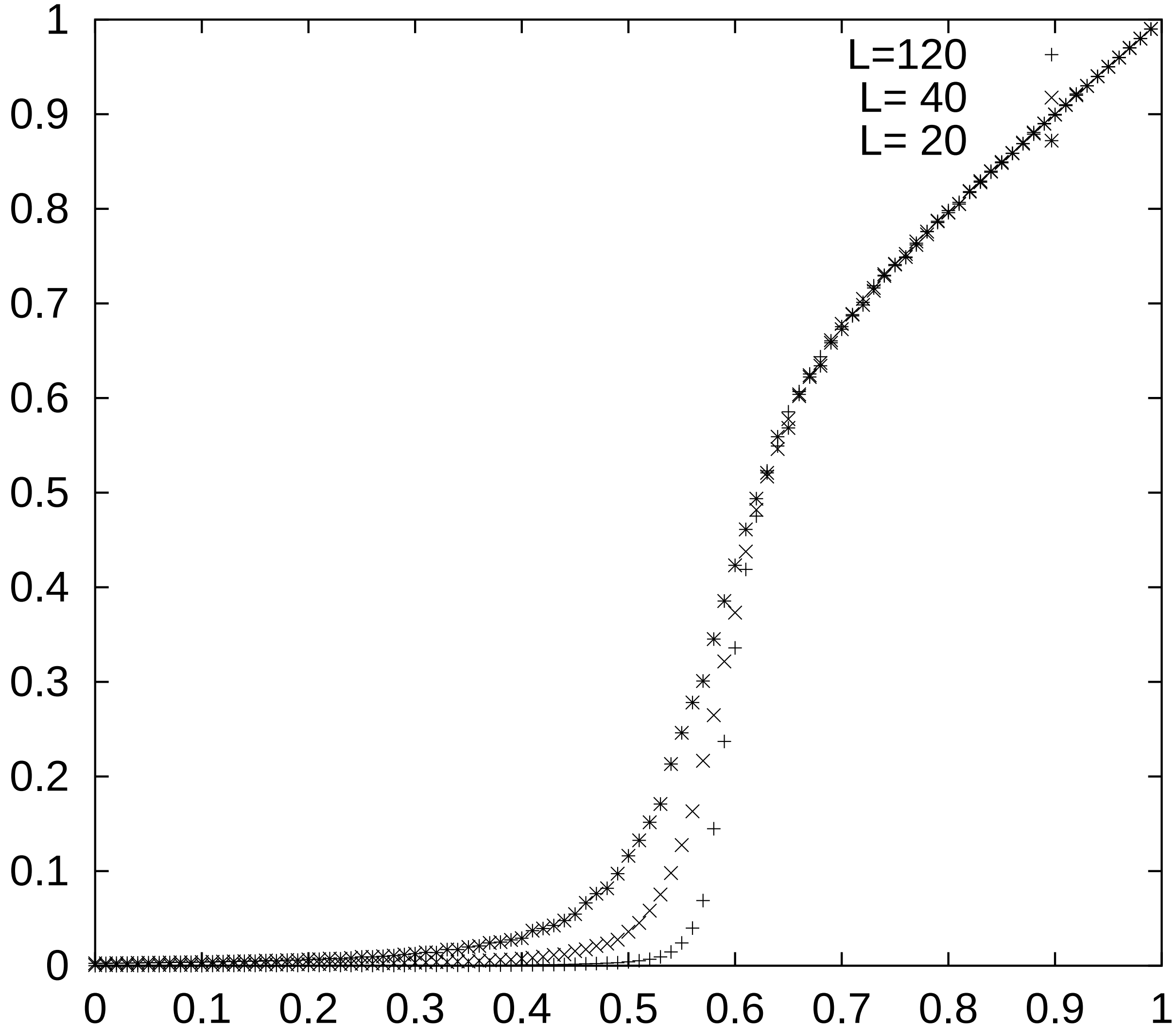
Suppose that buyers i occupying the nodes of a square lattice have different requirements p_i about the quality q of a product to be purchased. They don't know q a priori. They learn about it from neighbouring purchasers. Initially some agents purchased. An agent then decides to purchase if :

- at least one neighbour purchased
- the quality q of the product is larger than his private "preference" p_i.

Starting from an initial situation with very few purchaser, the amplitude of propagation of a purchase front depend strongly upon the density of possible purchasers. This dynamics of purchase is equivalent to the percolation problem known in physics, physico-chemistry, epidemiology...



q=0.5 q=0.55 q=0.6



Market share vesrsus fraction of would-be buyers

When q increases from zero, percolation theory predicts that purchase will remain zero until a critical density of possible purchasers and then roughly follows their number.

The transition occurs at the transition threshold $p_c$which value only depends upon the network chracteristics:

- $p_c = 0.593$ for a square lattice with 4 neighbours per site,
- $p_c = 0.407$ for a square lattice with 8 neighbours per site,
- $p_c = \frac{1}{k-1}$ for a random net.

The INCA model

Inhomogeneous Cellular Automata

Individual choices S_i ($S_i = 1$ buy, $S_i = -1$ don't buy) obey:

$$S_i = 1 \quad \textit{iff} \quad \sum_j S_j > \theta_i \tag{1}$$

Otherwise $S_i = -1$.

The sum is taken over the 4 agent's neighbours, and θ_i is the agent's individual threshold reflecting his requirements, propensity to buy according to some private info... It can be thought as a difference between the sellers public price p_s and the agents own reservation price p_i when the number of purchasers among his neighbours equals the number of non-purchasers.

$$\theta_i = p_s - p_i. \tag{2}$$

Predicting the dynamics in the extreme cases when all $\theta_i < -4$

(resp. $\theta_i > 4$) is easy: the attractor is 1 everywhere (resp. -1 everywhere)

For intermediate threshold values predictions are more difficult but the system is still well understood. Positive thresholds favour no-purchase global behaviour, and negative thresholds favour purchase behaviour, but which attractor is actually reached depends upon the details of the initial configuration.

This model describes many phase transitions in physics, such as crystal growth. The case of intermediate threshold values corresponds to spinodal decomposition when the presence of "seeds" allows the growth of the most stable phase.

Coupling adjustment with information contagion

Both "static" models predict phase transition in global purchase when the dynamics driving parameter takes values close to the threshold. It is then easy to predict that a seller who would try to adjust purchase accross the transition through price or quality would get into trouble.

Suppose that e.g. quality is above the threshold, resulting in large sales. If the seller reduces the quality to make larger profits, he would decrease the fraction of possible buyers (in the percolation model), cross the threshold downward and sales would plummet. (The INCA model predicts the same outcome).

To describe the coupled dynamics of adjustment and information contagion, let us use the INCA model and add that buyers re-adjust their reservation price p_i by some quantity $+dp$ (resp. $-dp$) after refusing to purchase (resp. accept to purchase). The symmetric adjustment of p_s by the seller would give equivalent threshold adjustment.

Algorithm:

- Start from a random distribution of choice S_i and thresholds θ_i;

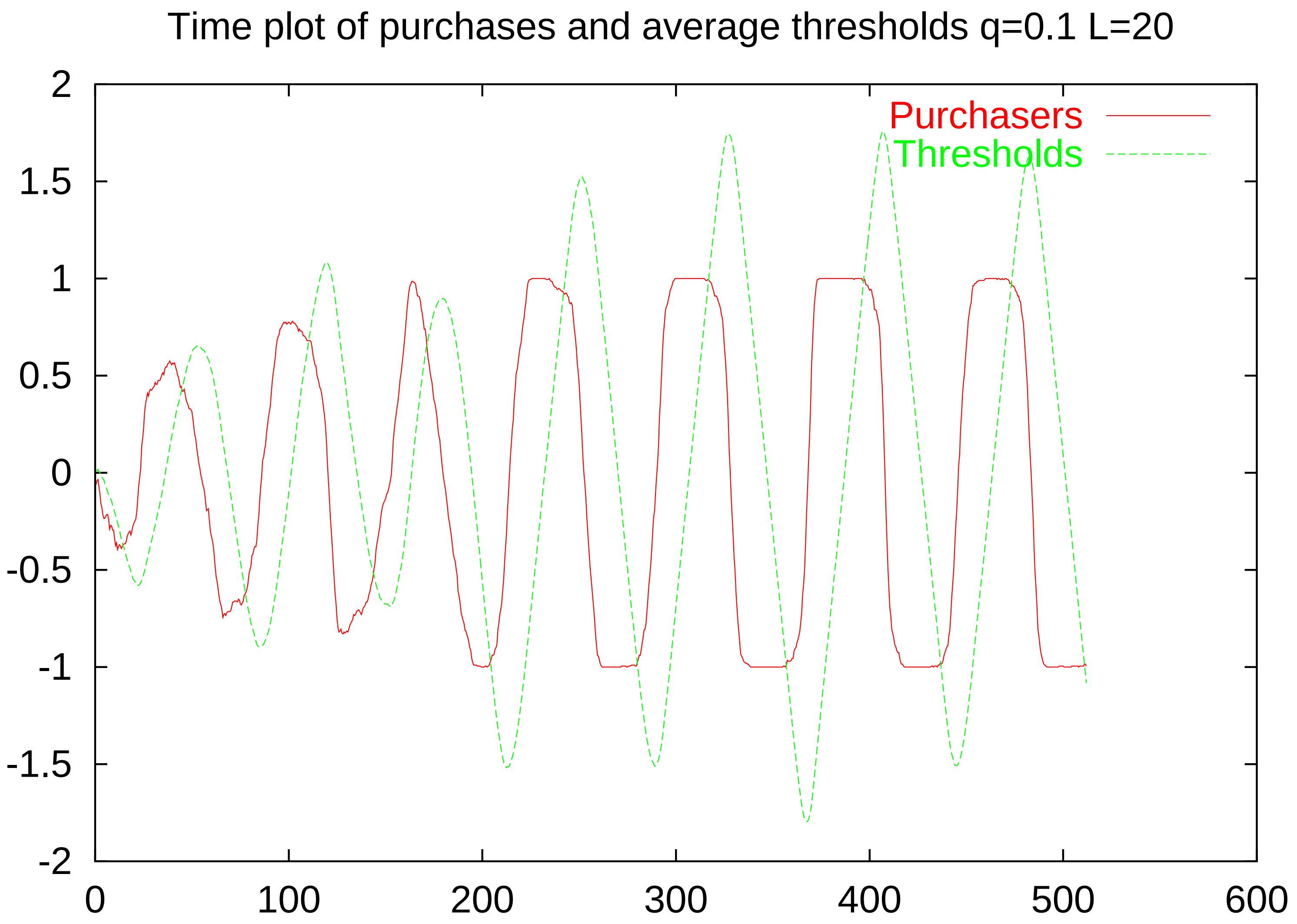
- At each time step, randomly choose a node i and update its choice S_i and thresholds θ_i according to the previous recipes (i.e. increase threshold by $+dp$ after purchase, decrease after refusal);

- Follow the global dynamics of purchase, thresholds, spatial correlation etc.

The observed global dynamics depends upon which process, adjustment or contagion is faster.

Slow adjustment dynamics

Corresponds to an adjustment time between the two extreme thresholds -2 and +2 divided by dp longer than the time necessary to sweep the lattice by opinion contagion $L/2$.

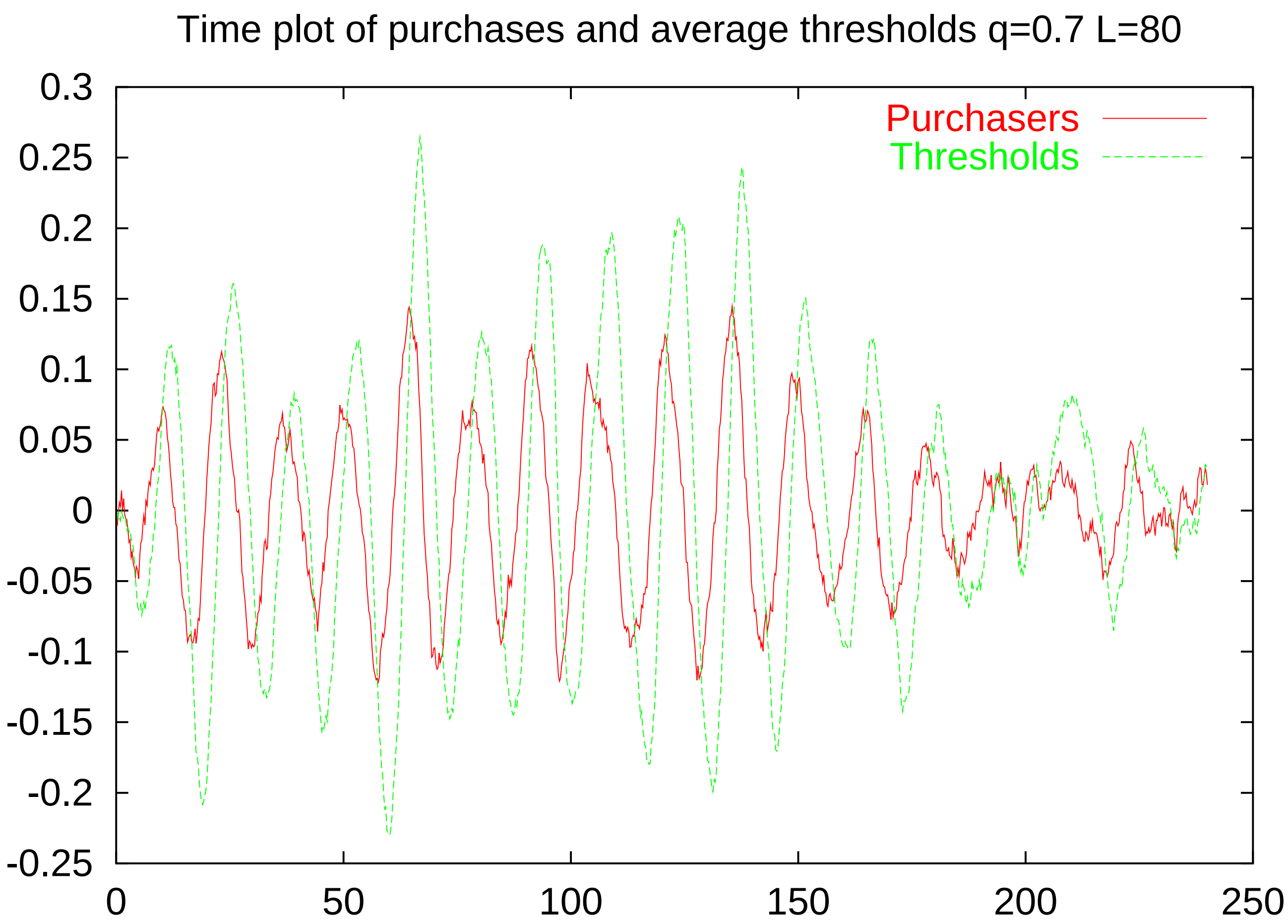


Time evolution of the average state of agents and average threshold, in the slow adjustment regime. (average state=1, everyone buys, average state=-1, no-one buys.). Unit time correspond to updating each site once on average).

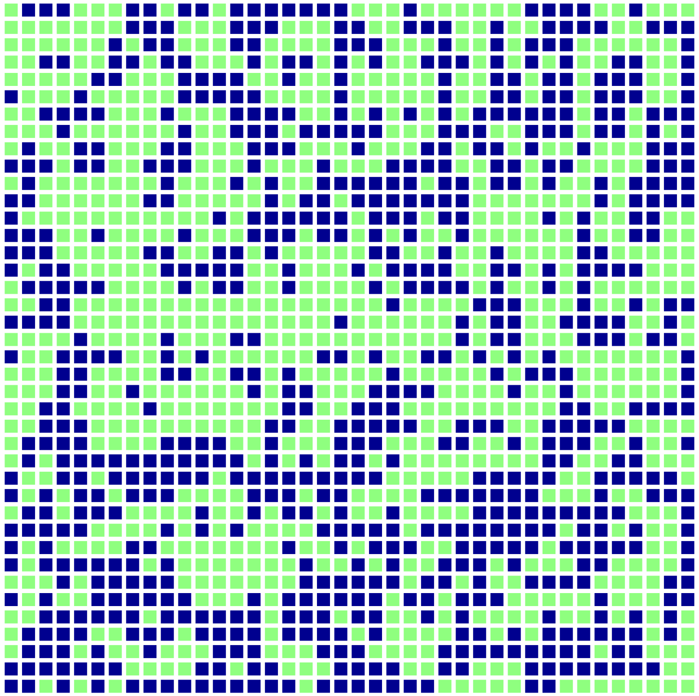
Once the lattice is in a saturated condition, say everyone buying, an isolated agent who would choose not to buy needs a threshold much higher than if she were surrounded by non-buyers. The system has to "wait" until thresholds which were low during the rise of the purchasing behaviour rise again to allow the apparition of isolated non-buyers. Hence the straight part of the average threshold evolution corresponding to its slow and regular increase. But as soon as isolated non-buyers are present, their neighbours need a lower threshold to switch to no-purchase; a wave of no-purchase propagates across the lattice.

Think of "business cycles".

Fast adjustment dynamics



Time evolution of the average state of agents and average threshold in the fast adjustment regime. Oscillation are smaller and less regular in amplitude than in the slow adjustment regime.



Pattern of behaviour at time 100, fast adjustment regime. Adjustment rate is 0.7. Green squares are buyers, black squares non-buyers.

Scaling

Periods T vary as:

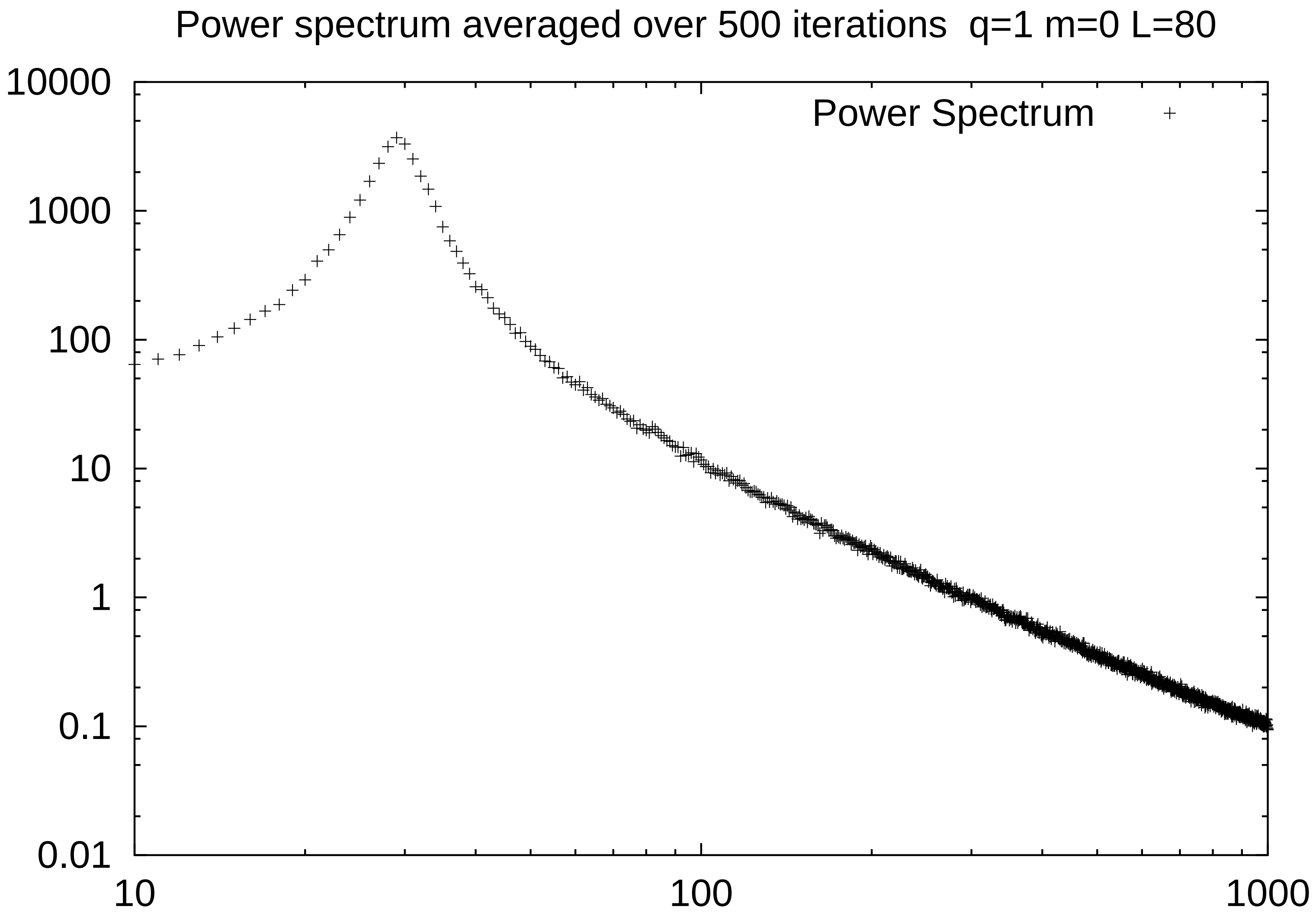
$$T \simeq \frac{10}{dp} \tag{3}$$

The period scales as average time it takes for threshold to switch between extremal values of -4 and 4. The threshold dynamics is the rate limiting step of the overall dynamics.

As seen in figure 3, amplitudes display a lot of variations. A simple way to average them on time is to measure power, namely the time averaged squared amplitudes. Average power scales as $L^2 = N$ the number of agents. If agents behaviour were oscillating in phase, we would expect power to scale in N^2. The scaling in N implies that N/s patches of constant size s oscillate independently giving:

$$P \sim \frac{N}{s} P_s \sim Ns \sim \frac{N}{q^2} \tag{4}$$

where P_s is the power of one patch, proportional to s^2. The scaling of s in q^{-2} is obtained from the equivalence between the time it takes for the social influence to sweep the patch and the time it takes to the threshold adjustment to sweep between the extreme values.



Power spectrum in the fast adjustment regime, for a large network

($L = 80$) and fast adjustment ($q = 1$) . The frequency scale correspond to 320 updating per agent on average for one frequency unit

Fourier power spectrum of the time series of agent states when $q = 1$. The large peak around abscissa 30 corresponds to a frequency of 10 iterations per agent. At larger frequencies, the long tail corresponds to a $1/f^2$ noise. Small scale correlations in agents behaviour due to local imitation processes are responsible for this long tail.

Conclusions

Most conclusions apply to both opinion contagion models (percolation and INCA).

- The models can be applied to a number of issues, after proper adjustments:
- Purchase on a market, adoption of new technologies, voting in the political arena;
- the propagation dynamics can be due to opinion exchange or positive externalities.

We use lattice topology because they are easily displayed, but results are directly generalised to other topologies such as scale free networks.

The scale free power spectrum of fluctuations is observed for percolation and INCA models.

The limit cycles observed for the INCA model in the limit of slow adjustment is a possible endogenous model for business cycles.