Agents-Based simulation and Multi-agent Systems

discrete choice with externality, network effects

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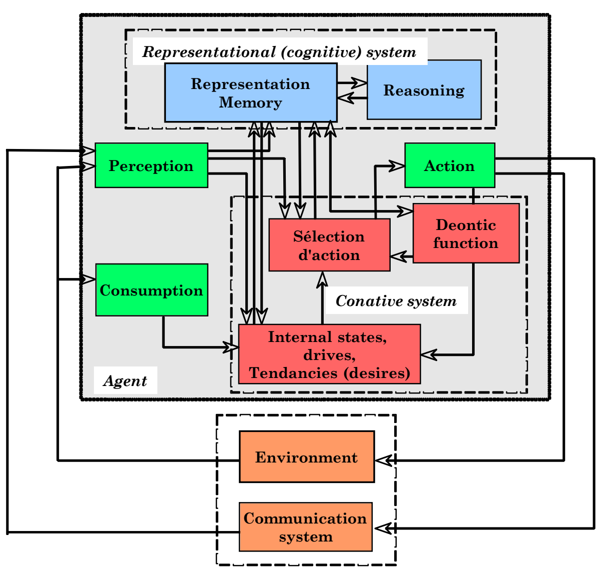
Agent-Based Model & Multi-agent systems

- Agent-Based Model (ABM) or ACE, (Agent-Based Computational Economics) are *strategy for modeling*. Models are based explicitly on Heterogeneous, agents generally interacting in a non trivial way, in order to produce global dynamics which differs of the simple aggregation of individual behaviors, i.e; ABM is generally a "*complex adaptive systems*".
- But most of ABM do not use "multi-agent systems" for the computer implementation (ie ABM implemented by the means of procedural programming (C, Fortan), mathematical Simulation Software (mathlab), cellular automata, or other way (Excel)
- Multi-agent systems is a software technology, using agent – oriented programming…

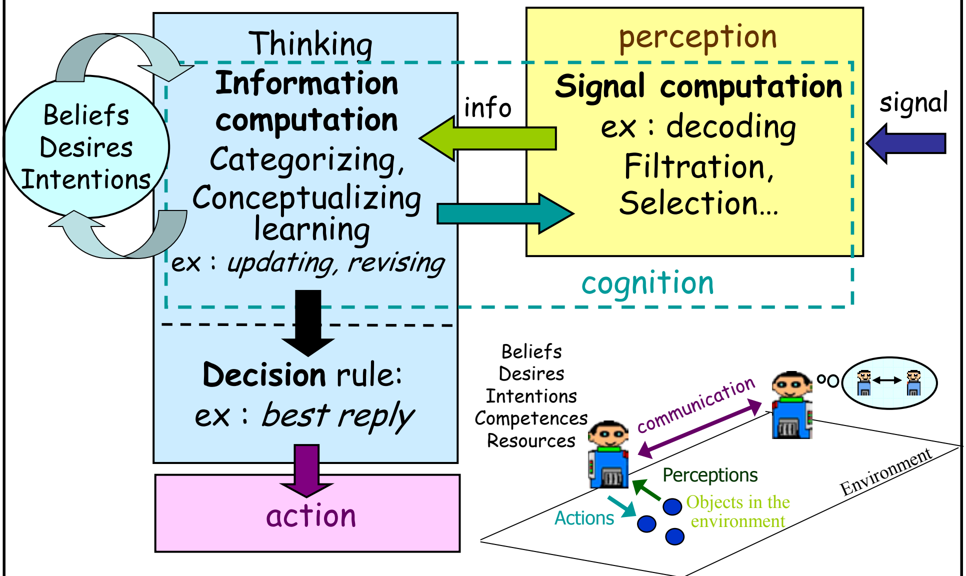
Agents as a part of Multi-agent system

- **Definition**: an agent is a software or hardware entity (a process) situated in a virtual or a real environment:
  - Which is capable of acting in an environment,
  - Which is driven by a set of tendencies (individual objectives, goals, drives, satisfaction/survival function),
  - Which possesses resources of its own,
  - Which has only a partial representation of this environment,
  - Which can directly or indirectly communicate with other agents
  - Which may be able to reproduce itself
  - Whose autonomous behavior is the consequence of its perceptions, representations and interactions with the world and other agents (fig. 1.1).

Agent architecture



The three dimensions of cognitive agent (perception –thinking / decision – action)



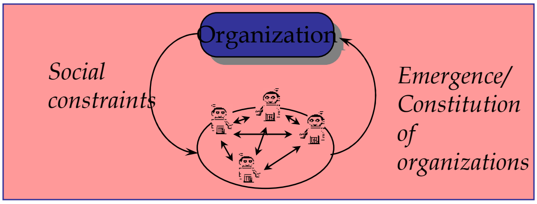
Cognitive Hierarchy (Dessalles, Muller, Phan, 2007)

1. Reactive Agent (RA) has a pre-determined response for a given state of its environment ("*situatedness*",).
   - Such an agent may represent both a material entity (such as a "spin" in a ferromagnetic set) or all living entities (pre-determined patterns of behaviour or "*Darwinian entities*" for Dennett, 1996).
2. Behavioural Agent (BA) may modify its behaviour for a given state of its environment according to the observed (historic) payoff from its past actions ("*consciousness*").
   - low cognitive rationality, but adaptability + "consciousness" [Bourgine 1993], [Phan 2004]. ("*Skinerian entities*", Dennett, 1996) reinforcement learning - e.g. Q-Learning, classifiers, etc.).
3. Epistemic Agent (EA) uses a model of its environment to pre-select actions ("*awareness*".)
   - medium or high cognitive rationality; In the hierarchy introduced by (Dennett, 1996) both *Popperian and Gregorian entities* are different cognitive levels of epistemic agents.

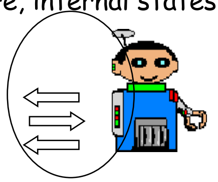
Axis for analysis (Ferber)

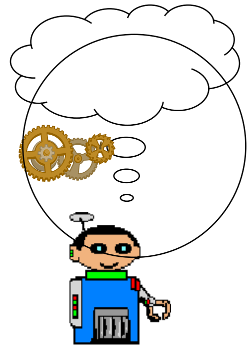
A multiagent system (as a social system) may be analyzed along two axis

- Axis: individual/collective



- Axis: external (behavioral) / internal (architecture, internal states)





Quadrants (Ferber)

Internal-Individual (I-I) **I (je)** *subjectivity* <mental states, emotions, beliefs desires, intentions, cognition > Interiority	**External-Individual (E-I)** **it, this** *Objectivity* <agent behavior, object, process, physical entities > Observables, exteriority
Internal-Collective (I-C) **We** *intersubjectivity* <shared / collective knowledge invisible social codes and implicit ontologies, informal norms and conventions > Noosphère	**External-Collective (E-C)** **Them, all this** *Interobjectivity* <reified social facts and structures, Organizations, institutions> Sociosphere (social structures)

Why simulated ? Agent-Based simulation and experimentents

Axelrod (2005) : the major role of discovery (*experiments*)

- As a scientific methodology, simulation's value lies principally in *prediction*, *proof* and *discovery*
- Simulation is "a third way of doing science".
- Like deduction, it starts with *a set of explicit assumptions*.
- But unlike deduction, *it does not prove theorems*.
- Instead, a simulation generates *data* that *can be analysed inductively (quasi-empiric experiment)*.
- Unlike typical induction, however, the simulated data comes from rigorously specified set of rules rather than direct measurement of the "real world".
- Simulation modelling can be used as an *aid to intuition*.
- Simulation is *a way of doing experiments*
  ➔ *Serendipity (Merton*)

KISS-"Keep It Simple & Stupid" (Axelrod 2005,p.6)

- "The goal of agent-based modelling is *to enrich our understanding of fundamental process* that may appear in variety of applications.
- This require adhering to the KISS principle, wich stands for the army slogan "*Keep It Simple & Stupid*"
- The Kiss principle is vital because of the character of the research community.
- Both the researcher and the audience have limited cognitive capacity.
- When a *surprising result* occurs (*serendipity*), it is very helpful to be confident that one can understand everything that went into the model.
- Simplicity is also helpful in giving others researchers a realistic chance of replicating one's model, and extending the work in new directions.

Magritte

Models

- Model as the *interpretation* (syntactic) or one of the possible interpretation (semantic) of a preexisting theory.
- History of sciences: models are often designed for to resolve a practical (engineering) question, without necessarily pre-existing theory
- Pragmatic definition of (Minsky, 1965): « *To an observer B, an object A* is a model of an object A to the extent that B can use A* to answer questions that interest him about A* ».
- "*model building*" activity, isolates and analyzes specific dependencies taking from empirical phenomenon, focusing on few factors and excluding everything else with as objective "to understand how just these aspects of reality work and interact" [Solow 1997].
- Models as *Mediator* (Morgan, Morrison, 1999) or as *explanatory Metaphor* (McClosey 1983)
- Models are *conceptual exploration* or *quasi* (thought) *experiment* (isolation method or models are "economist's laboratories" Mäki, 1992, 2002)
- Model are *credible possible world* (Sugden, 2002)

Bifurcation, plurality of attractors and non trivial global behavior : the example of the simple model of social influence by Orlean (1998) – a continuous state version of Banerjee's (quoted by Alan)

a simple model of Social Influence (A. Orléan)

- the « world » can be in two states : {0,1}
- At each period, one agent is taken at random (asynchronous scheduling). He get a *private information on the* « state of the world ».
- Let theta the probability that this information was 1.
- If this i *private information is coherent with public information* (the % of the population that believe that the stae of the world is 1), This self enforce his opinion.
- If the two information are contradictory there is a « *cognitive dissonance* ». In that case, parameter mu gives the probability for that agent follows the public opinion.
- (mu can be view as the confidence of the agent in the reliability of his own information, but it is not the only possible *interpretation)*
- If mu is close that 1, agents are mimetics, if mu = 0 agents follow their private information only

Le modèle simple d 'Orléan (1998,2002) (a)

States of world {0,1}; Private Signal σ; public information η

$$\text{proba}(\sigma_i = 1 | M = 1) = \text{proba}(\sigma_i = 0 | M = 0) = p$$

$$\text{proba}(\sigma_i = 0 | M = 1) = \text{proba}(\sigma_i = 1 | M = 0) = 1 - p$$

- « cognitive dissonance ». Agents follow public information with probability μ, eand their private information with probability $1-\mu$.
- $q_\mu(\sigma,\eta)$: probability that one individual with coefficient of mimetism μ chooses the state [1] when he observes information corresponding at η with a private signal σ.

$$\text{si}\,\eta < 0{,}5 \qquad q_u(1,\eta) = 1-\mu \qquad q_u(0,\eta) = 0$$

$$\text{si}\,\eta > 0{,}5 \qquad q_u(1,\eta) = 1 \qquad q_u(0,\eta) = \mu$$

Transition probability is the state of the world is M=1 :

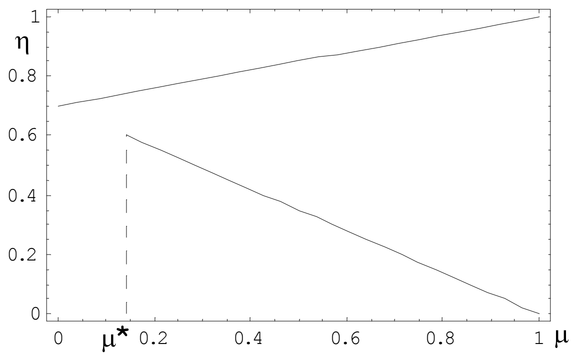
$$p^1_{0\to1}(\eta) = P^1(\eta) = p.q_\mu(1,\eta) + (1-p).q_\mu(0,\eta)$$

Simplest model of Orléan (1998,2002) (a)

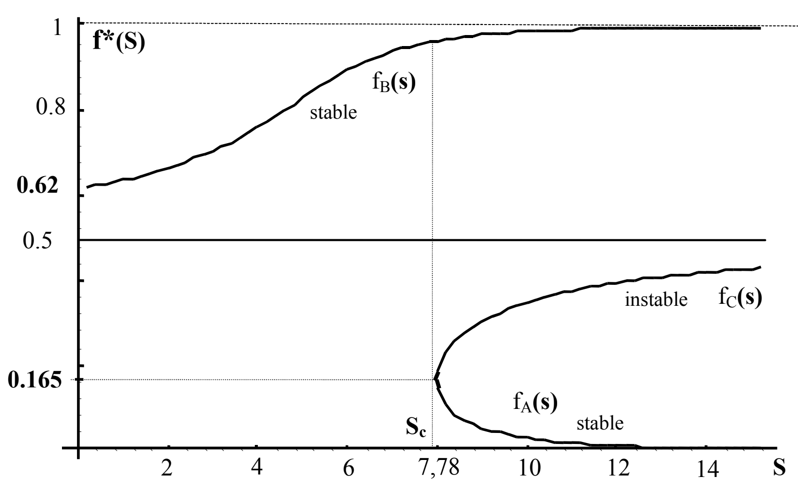
- resolution of $P^1(\eta) = \eta$ gives two regimes separated by a critical value:
- (graphical representation : mode(s) of the asymptotical invariant distribution)

$$\mu^* = (2.p-1)/2.p$$

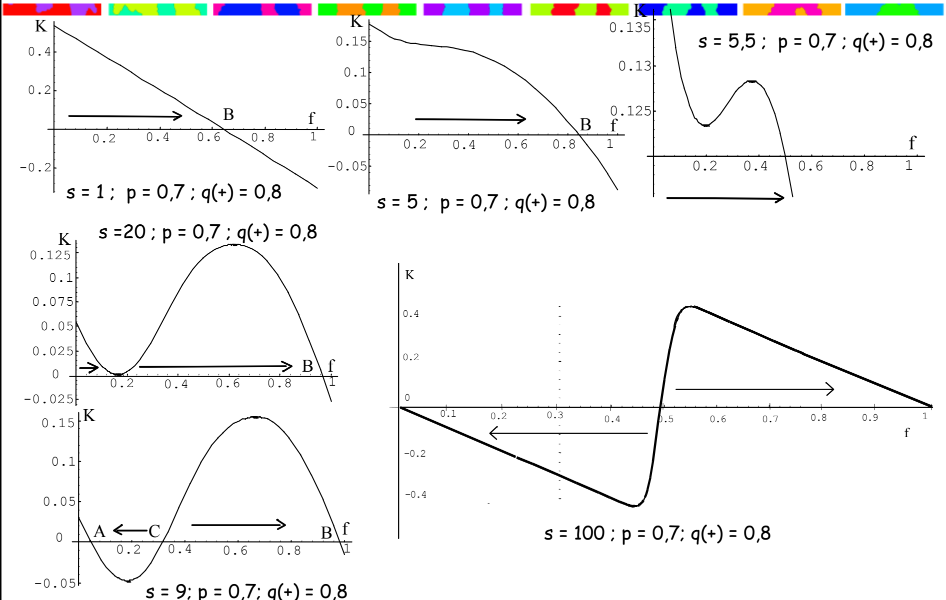
$$\eta_{s+} = p + (1-p)\mu \qquad \eta_{s-} = (1-\mu)p$$



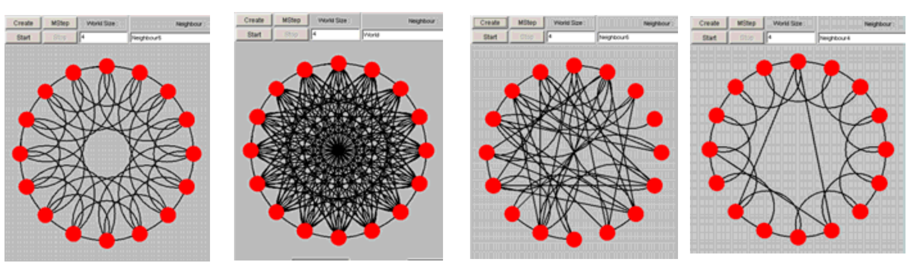
Orléan (1995)



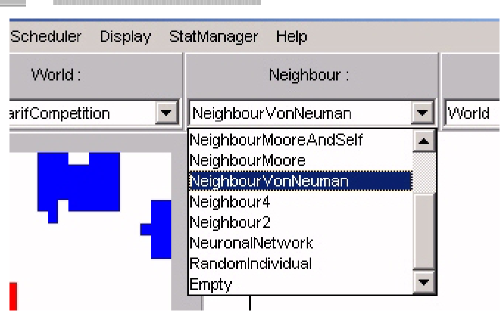
Orléan (1995)



How to simulate ? *Computational Laboratory*, a friendly and intuitive way for manipulate networks



CL provides "a clear and easily manipulated graphical user interface that can permit researchers to engage in serious computational research even if they have only modest programming skills" (Tesfatsion, 2002)

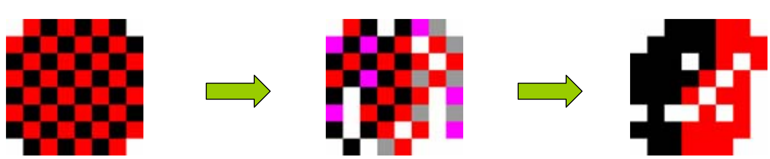


Dealing with « structures » : Interactions and emergent properties

Emergence in multi-agent systems : what does emerge in Schelling's model of spatial segregation?

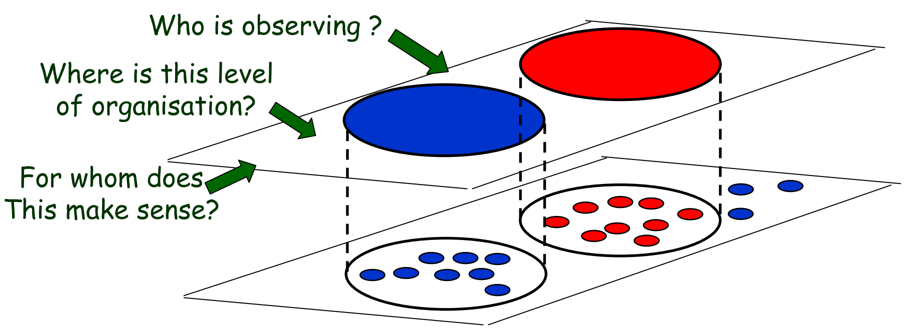
- Schelling's aim was to explain how segregationist residential structures could spontaneously occur, even when people are not segregationist themselves.
- Agents interact only locally with their 8 direct neighbours ("Moore'' neighbourhood). *No global representation* about the residential structure *is available to agents*.
- Each agent would stay in a neighbourhood with up to 62% of people with another colour

- A slight perturbation is sufficient to induce a local *chain reaction* and the *emergence* of segregationist patterns - «segregation » (clusters) is an *emergent propriety of the model…*

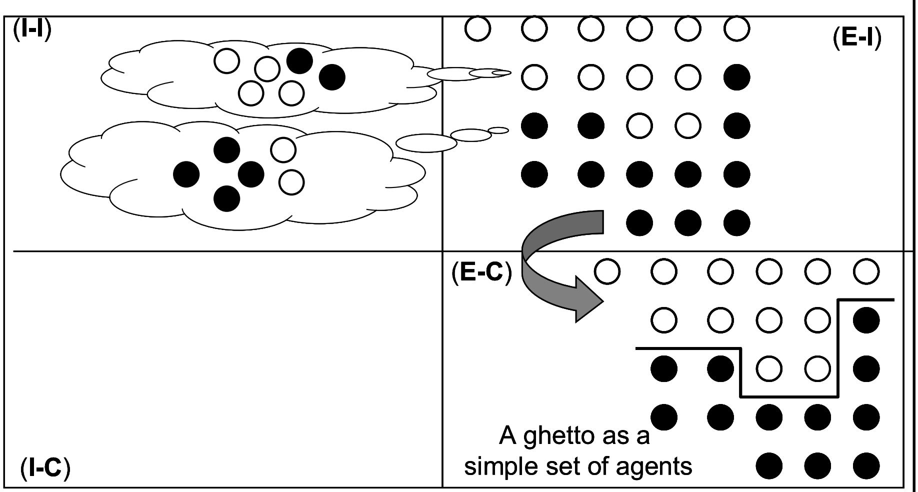


The Schelling model of Segregation : a case of ontological emergence ?

- Local interactions are sufficient for spatial homogeneous patterns to occur: spatial segregation is an emerging property of the system's dynamics, not an attribute of the individual agents.
- What is the ontological status of the "levels of organisation" and of the "observer" ? What is the semantics of the emergent phenomenon ?



The methodological individualist approach: weak emergence of ghettos in Schelling's model of segregation



Methodology MAS as complement or substitute

Start: Empirical question > model building (Solow, 1957)

Step 1:c*onceptual exploration* (Hausman 1992)
formalizing, implementing, evaluating…in the « world of model »

Mathematical Methods (MM) (analytical solution)	Classical simulation procedural programing	MAS object programming MAS as simple network of automata

Step 2: systematic **exploration** when **there is no analytical** solution ex: non trivial local interactions, simple learning (updating)

Model to Model | *Benchmarking, finite size effects* | *exploration..*

Classical Monte-Carlo simulation procedural programing	MAS agent (object) programming MAS as sophisticated network of automata

MAS as complement o MM

Step 3: systematic virtual **experiments** on more sophisticated cases (inter-related models) ie : multi-scales, multi-perspectives MA Systems; cognitive & social agents; no unified model, but ecology of co-evolving entities (*quasi-empirical AS*)

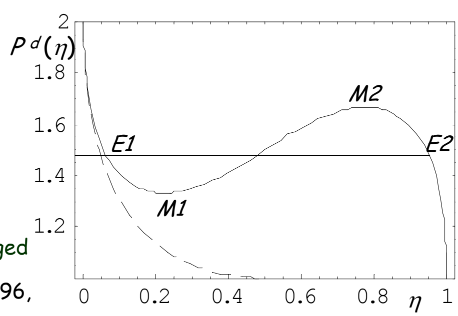
MAS agent (object) programming
With cognitive, communicative, social agents

MAS as substitute of MM

(1) Empirical question : Individual decision in a social context : related literature

- In many circumstances, Economists and Sociologists have underlined the aggregate consequences of Social Influence upon individual decisions:
  - Bandwagon effect (Veblen, Leibenstein 1950, Simon 1954), Telecommunication services: network and consumption externality (Rabeneau, Sthal 1974, Curien Gensollen 1987, Rohlf 2002) and others forms of decision with social influence (Schelling, 1973, 1978, Granovetter 1978, Becker 1974, 1991, De Palma Lefebvre 1983…)
  - The relationship between the *distribution of individual idiosyncratic characteristics, the strength of social influence and global behavior* is not clearly established in a general setting. The role of local influence is underlined by Rolf (1975) and Valente (1995) among others, but not resolved.

In road for a more general setting:
Pioneers: Random (walrassian) economies with social influence (Gibbs Fields) Föllmer (1974) ;
Ising models: Galam, Gefen, Shapir 1982
Recents contributions start with Durlauf's work at the beginning of 90's and Blume (1993, 1995) "paradigmatic" example is Brock-Durlauf 2001 (*RES*)
Link with statistical mechanics is acknowledged Blume (1993, 1995), Durlauf, 1997, 1999 see also: Glaeser, Sacerdote, Scheinkman 1996, Glaeser, Scheinkman 2002….
Local influence Elisson (1993) Ioanides (2006)
➔All these models have RANDOM UTILITY



Becker (1991) "A Note on Restaurant Pricing and Other Examples of Social Influences on Price", *Journal of Political Economy*, 99(5) 1991 p.1109-1116.

the Generic framework of the GNP model heterogeneous (fixed) IWA and social influence

- A population of N agents ; Each agent i has to make a binary choice: to adopt /buy ($\omega_i=1$) or not ($\omega_i=0$)
- Each customer's willingness to adopt (IWA) is the sum of two terms:
- An idiosyncratic term H_i randomly distributed in the population, but remains fixed (H : mean value of the distribution)
- A social influence term: a weighted sum of the choices of other agents

$$H_i + \frac{1}{\|\vartheta_i\|} \sum_{k \in \vartheta_i} J_{ik} \omega_k$$

Idiosyncratic willingness to adopt (H_i)

weight given by agent i to the choices of his neighbours (J_{ik})

choice of neighbour k (0 or 1) (ω_k)

number of « neighbours » of i ($\|\vartheta_i\|$)

« neighbourhood » of i (ϑ_i)

- strategic complementarity: $J_{ik} > 0$
  - making the same choice as the others is advantageous
- homogeneous social influence ($J_{ik} = J$) :

$$J \frac{1}{\|\vartheta_i\|} \sum_{k \in \vartheta_i} \omega_k = J\, \eta_i$$

weight of neighbours' choices (J)

fraction of i's neighbours that adopt (η_i)

BDD versus GNP (noiseless) model

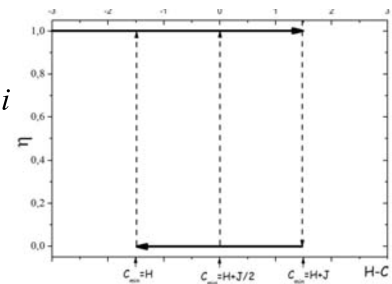
- BDD: (Durlauf 1997, Blume, Durlauf 2001, 2003, Brock Durlauf 2001a, 2001b…) use the Random Utility Model (Thurstone 1927, Luce 1959) + Quantal Choice Analysis (Luce, Supes 1965, McFadden 1974)
  - Each alternative random term is i.i.d. double exponential (extreme value type I) distributed > probabilistic choices: the join distribution of choice is logistic
  - Idiosyncratic heterogeneity concerns the random term
  - "*Classic Ising Model with annealed disorder*"
- GNP: (Nadal *et al.* 2003, Gordon *et al.* 2005; Nadal *et al.* 2005, Phan, Pajot 2006, Gordon *et al.* 2006, Phan, Semeshenko 2007)
  - The heterogeneity concerns the fixed, but Idiosyncratic Willingness to Adopt (pay): IWA
  - No stochastic term, random IWA is "quenched" > deterministic maximization
  - "*Quenched Random Field Ising Model*" (quenched or frozen disorder)

Adoption & Hysteresis

- Agent choice to maximize:

$$W_i(\eta_i) = S_i(\eta_i)\omega_i = \omega_i(H_i - C + J\eta_i)$$

- Where C is an exogenous cost (price, in the market case)
  - Direct adopter: $H_i > C$
  - Indirect adopter: $H_i + J.\eta_i > C$
- Homogeneous case: $H_i = H$ for all i
  - Two neighbors
  - Full connectivity
  - "static" hysteresis



- Back to the heterogeneous case: H_i →

Analogy with game theory (bounded distribution)

Payoff column nr

G1	$\omega_{nr}=0$	$\omega_{nr}=1$
$\omega_i=0$	0	0
$\omega_i=1$	$H_i - C$	$H_i - C + J$

row i

↔

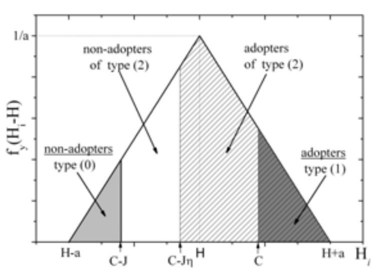
Cost of unilateral deviation

G2	$\omega_{nr}=0$	$\omega_{nr}=1$
$\omega_i=0$	$C - H_i$	0
$\omega_i=1$	0	$H_i - C + J$

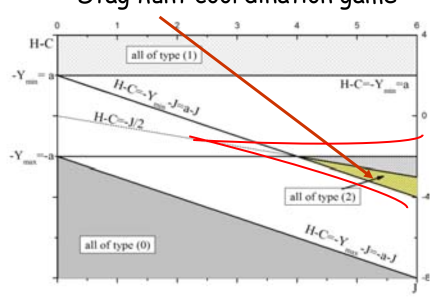
Potential game

Dominance anaysis: types of agents

- type 0: never adopt if $H_i + J < C$
- type 1: always adopt if $H_i < C$
- type 2: adopt if $H_i + J\eta < C$



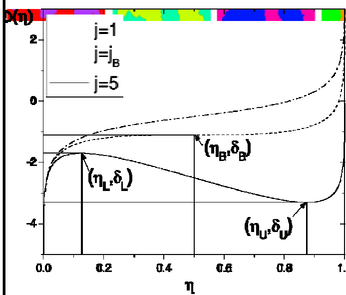
Stag hunt coordination game



Full connectivity
Nash equilibria
GNPV (2006)

$$\eta = P(H_i > C - J\eta) = \int_{C-J\eta}^{\infty} f(H_i)\,dh$$

Global externality: Uniqueness or multiplicity of equilibrium(s)



(1) $s \equiv \delta + j\eta$

with: $\delta = h - p$

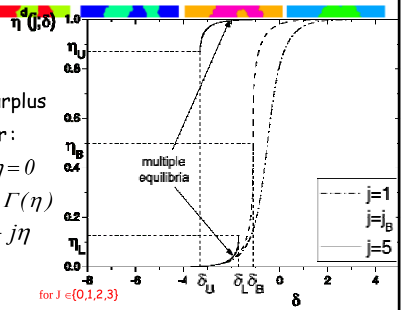
s: average ex-ante surplus

s: marginal customer:

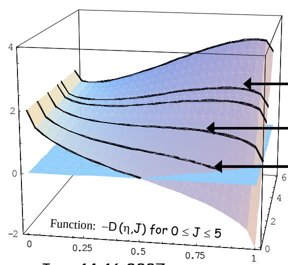
(1 bis) $(h - s) - p + j\eta = 0$

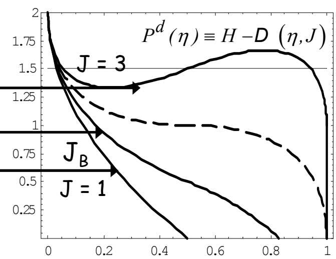
(2) $\eta = 1 - F(-s) \Leftrightarrow s = \Gamma(\eta)$

$\delta = D(\eta) \equiv \Gamma(\eta) - j\eta$



•Relationship with the "classical" downward-slopping demand curve (Becker 1991)





Uniqueness :

Moderate Social Influence

Assumption

Glaeser, Scheinkman, (2002)

Multiplicity :

Gordon et al. (2006)

Sethna's hysteresis in local networks: multiplicity of equilibria, path dependence, homogeneous domains (clusters) return point memory effect

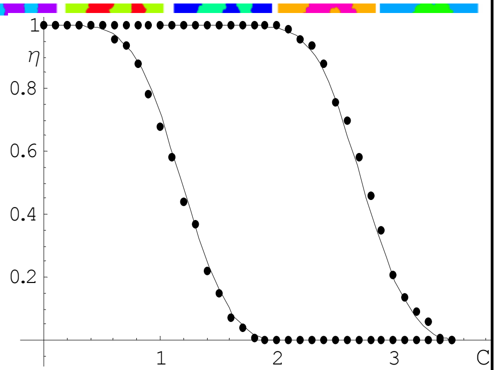
Analytical and Simulation results

(preliminary investigations)

Starting from a homogeneous state without adoption to complete adoption and return: the larger hystersis loop

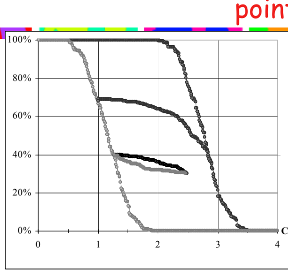
"Validation" of the computer implementation:
Theoretical infinite dimensional network (Skula 2000) vs finite dimension ACE network

$$P_0 = P(H_i > C) = P(type\ 1, \omega_i{=}1\ if\ \eta = 0)$$
$$P_1 = P(H_i > C - J/2) = P(\omega_i{=}1\ if\ \eta = 1/2)$$
$$P_2 = P(H_i > C - J) = P(\omega_i{=}1\ if\ \eta = 1)$$
$$P_1 - P_0(C) = (C > H_i > C - J/2) =$$
$$P(to\ be\ of\ type\ 2\ AND\ \omega_i{=}1\ if\ \eta = 1/2)$$
$$with\ P_i \equiv P_i(C)$$
$$P(\omega_i = 1|C) = P(\eta_i = 1).P_2 + P(\eta_i = 1/2).P_1 + P(\eta_i = 0).P_0$$
$$with: \quad \eta(C) = P(\omega_i = 1|C)$$
$$P(\eta_i = 1) = P(\omega_{i\pm 1} = 1|C, \omega_i = 0)^2$$
$$P(\eta_i = 1/2) = 2.P(\omega_{i\pm 1} = 1|C, \omega_i = 0)P(\omega_{i\pm 1} = 0|C, \omega_i = 0)$$
$$P(\eta_i = 0) = P(\omega_{i\pm 1} = 0|C, \omega_i = 0)^2$$
$$P^*(C) \equiv P(\omega_{i\pm 1} = 1|C, \omega_i = 0) = \lim_{m\to\infty} P_0 \sum_{k=0}^{m} [P_1 - P_0]^k = \frac{P_0}{1 - [P_1 - P_0]}$$
$$[1 - P^*(C)] \equiv P(\omega_{i\pm 1} = 0|C, \omega_i = 0) = \frac{1 - P_1}{1 - [P_1 - P_0]}$$

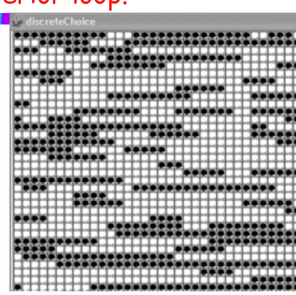


Theoretic and simulated values (dot) for the main hystersis
for: N = 1156 agents, $N\vartheta$ = 2 (circle); J = 4: H=0;

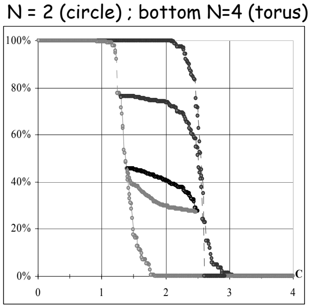
The inner hysteresis loop: reversing the Cost from an arbitrary point on the exterior loop.

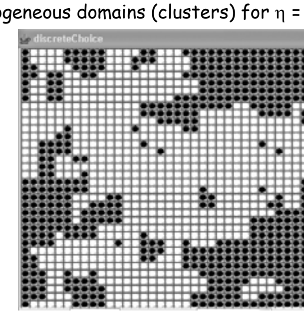


Sethna's inner hysteresis J=4,
top: N = 2 (circle) ; bottom N=4 (torus)



homogeneous domains (clusters) for η = 40% C=1.41

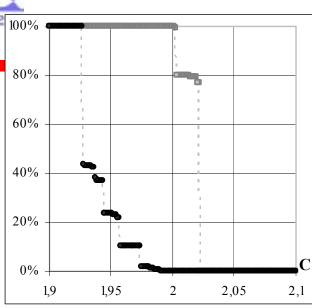




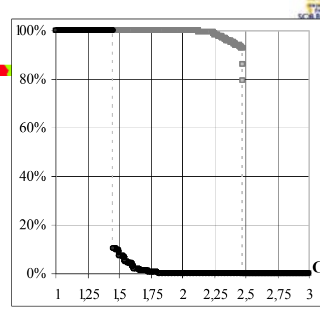
Sethna's hysteresis in local networks:Sum up

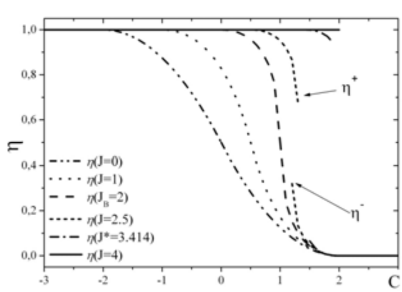
- The GNP model (binary choice with externality among heterogeneous agents) has hysteresis property (two stable equilibria) for "sufficiently" strong social influence and weak dispersion of IWA
  - (Nadal *et al.* 2003, Gordon *et al.* 2005; Nadal *et al.* 2005, Phan, Semeshenko 2006)
- Calculus based upon method introduced by (Shukla, 2000) allow us to *calculate exactly the hysteresis path* for N=2 both starting from an homogeneous state (nobody adopt) or from an arbitrary initial state.
- Simulations conducted on a multi-agent platform "Moduleco-Madkit" allow us to experiment the extension of this theoretical results to numerous networks configurations
- For a given value of the external parameter (e.g. price, cost), there is a multiplicity of equilibria, depending on the previous state of the system (path-dependence).
- If this parameter returns back to some initial value, *the system returns precisely to the same state from which it left*. The inner loop illustrates the return point memory effect, in which the system remembers its former state.

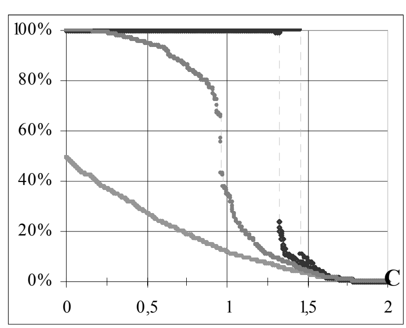
Hysteresis



increasing H-C
decreasing H-C
N=1156, J=4
synchronous dynamics
Step 10^{-4}







Methodology MAS as complement or substitute

Start: Empirical question › model building (Solow, 1957)

Step 1:c*onceptual exploration* (Hausman 1992)
formalizing, implementing, evaluating…in the « world of model »

Gordon *et al*, 2006, 2007 (gal case)

Phan, Shemeshenko 2007
(analytical solution)

Phan Pajot 2006

Phan Pajot 2006
Phan 2007

Model to Model | *Benchmarking, finite size effects* | *exploration..*

Step 2: systematic **exploration** when **there is no analytical** solution *MAS as complement o MM*

Nadal et al, 2005
Shemeshenko *et al* 2007
(EWA learning)

Phan, Pajot Nadal, 2003
Phan 2007

Step 3: systematic virtual **experiments** on more sophisticated cases (inter-related models) ie : multi-scales, multi-perspectives MA Systems; cognitive & social agents; no unified model, but ecology of co-evolving entities (*quasi-empirical AS*)

Dessalles Phan 2005, 2006
Phan Ferber 2007
(disconnected)

MAS as substitute of MM

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Any Questions ?