Computational Mechanism Design

Jasmina Arifovic

September 18, 2007

Introduction

- joint work with John Ledyard, Caltech
- research project - attempt to create a valid computational testbed for mechanism design
- a collection of artificial agents that will provide results similar to those from laboratory experiments with human subjects
- if a testbed is to be at all useful, it must match laboratory performance over a wide range of environments and a wide range of mechanisms
- Arifovic and Ledyard (2003) - a public goods environment and a class of Groves-Ledyard mechanisms
- We were able to create a valid testbed for that situation.

Private Goods

- evaluate market designs in several private goods environments.
- Call Markets
- One good, one market, heterogenous buyers and sellers
- Extensions: multiple goods, multiple markets
- additive and quadratic utility

Computational Mechanism Design

- Market design - many questions and choices: e.g., continuous auction or call market, open or closed book etc.
- 3 scientific approaches: theoretical, experimental and computational
  - Theoretical: rational agent, game-theoretic framework, for example, work on call markets - Satterthwaite and Williams (1989)
  - Experimental: Human model - subjects in the experiment, examples for call markets - McCabe, Rassenti, and Smith (1993), Friedman (1993)
  - Computational mechanism design: model of the agent is a computer program, a "smart" agent

The Testbed

- A testbed consists of a collection of computer agents ready to respond to a variety of mechanism designs in a variety of environments
- It must be flexible enough to take in whatever information we provide about e and (M, g, s) and produce whatever messages are required.
- Not necessary or desirable to create agents that are smarter or faster than humans
- We need to create agents who produce outcomes similar to those produced by human agents when placed in the same environment
- Experiments with human subjects - test of success

Individual Evolutionary Learning

- At the beginning of round $t \in \{1, 2, ..., T\}$, each agent $i \in [1, \ldots, N]$ has a collection A^i_t of possible messages consisting of J alternatives $a^i_{j,t} \in A^i_t$, $j \in \{1, \ldots, J\}$
- At each t, an agent selects an alternative, m^i_t, randomly from A^i_t using a probability density π^i_t on A^i_t.
- The outcomes and feedback are then determined using the outcome function, $g(m_t)$, and the feedback functions, $s^i_{t+1} = s^i(m_t)$.
- Finally, using the information in s_{t+1}, each agent then computes a new A^i_{t+1} and π^i_{t+1}.

Updating of A_t

A_{t+1} - foregone utilities, replication, and experimentation:

- *Foregone utility* for each alternative in the set - the (expected) payoff, given the signal s_t^i, that the alternative $a_{j,t}^i$ would have received had it been actually used, taking the behavior of other agents as given
- *Experimentation* - For each entry in A_{t+1}^i and for each $j = 1, ..., J$, with probability ρ we select one message at random from M and let $a_{j,t+1}^i$ equal that message.

Updating, cont.d

- *Replication* occurs in a way that reinforces messages that would have been good choices in previous rounds
  - For $j = 1, \ldots, J$, pick two members of A_t^i randomly (with uniform probability) with replacement. Let these be $a_{k,t}^i$ and $a_{l,t}^i$. Then

$$a_{j,t+1}^i = \left\{ \begin{array}{c} a_{k,t}^i \\ a_{l,t}^i \end{array} \right\} \mathit{if} \left\{ \begin{array}{c} U(a_{k,t}^i|s_t) \geq U(a_{l,t}^i|s_t) \\ U(a_{k,t}^i|s_t) < U(a_{l,t}^i|s_t) \end{array} \right\}.$$

Updating, cont.d

- *Selection probabilities*

$$\pi^i_{k,t+1} = \frac{U(a^i_{k,t+1}|s_t)}{\sum_{j=1}^{J} U(a^i_{j,t+1}|s_t)} \tag{1}$$

for all $i \in \{1, \ldots, N\}$ and $k \in \{1, ..., J\}$.

Some remarks

Initialization:

- could be done in many different ways
- attempt to represent what subjects bring to the lab or represent thinking hard about the problem prior to the start
- we forego any attempts at sophisticated initializations and construct the initial set A_1^i by randomly selecting, with replacement, J messages from the set of possible messages, M (1/J)

Some remarks, cont.d

Replication for $t+1$

- favors alternatives with a lot of replicates at t and alternatives that would have paid well at t if they had been used.
- Over time, alternatives that consistently earn higher foregone payoffs receive more replicates and their prominence in the set increases.
- On the other hand, alternatives with consistently low foregone payoffs receive smaller and smaller number of replicates. Eventually, they disappear from the set
- The potentially successful alternatives are remembered and reinforced while the less successful ones are forgotten
- Over time, sets become more homogeneous as most alternatives become replicates of the best performing alternative.

Some Remarks, cont.

Experimentation

- Introduces new alternatives that may be tried out, in spite of their prior evaluations
- This insures that a certain amount of diversity is maintained
- Experimentation is not as random as it may look
  - an alternative is selected at random from M
  - but, it must have a reasonably high foregone utility relative to the last period or future periods to have any chance of ever being used
  - it has to increase in frequency in order to increase its selection probability; this can happen only if it proves successful over several periods

Call Markets

- **Call market** - accumulates bids and offers from traders for a period of time and then, in a batch process, clears them at a uniform price.
- repeated environment - traders participate in a series of call markets in the same environment
- the effect that different information treatments have on allocative efficiency, after each call
    - *open book* - information about all the bids and offers that were processed in that call
    - *closed book* - the clearing price and whether they traded or not

Overview of the Results

- *providing information about the book to traders between calls lowers allocative efficiency*
- information creates noise
- result of the alternative designs of the markets or result of the particular algorithm that we are using
- we ran a series of economics experiments that replicated both the environments and the market designs
- allocative efficiencies produced in the laboratory experiments were similar to those produced by our agents, thereby validating our computational approach.

The Environment

- a fixed number of buyers, N, and sellers, N.
- Sellers each own one unit of a commodity and buyers each want to consume one unit of the commodity.
- Sellers must pay a cost if they sell, buyers receive a value if they buy.
- Each buyer's valuation of a good is given by $V_i \in [0, \eta]$, where $i = \{1, \dots N\}$
- Each seller's cost of a good is given by $C_j \in [0, \eta]$, where $j = \{1, \dots N\}$.
- Each buyer i knows V_i and N and each seller j knows C_j and N - this is common knowledge.
- we test two versions of a call market.

A Simple Call Market

- There is only one call before trade occurs and there is no information revealed about submitted bids or offers before the call is made.
- The simple call market, SCM, is a sealed-bid auction in which buyers submit bids, b^i, and sellers submit offers, o^i, to a "market".
- When all bids and offers are collected, the market is "called". The market computes a clearing price.

Market Clearing Price

- First, all bids and offers are ranked:
- Let $b^1 \geq b^2 \geq ... \geq b^N$ and $o^1 \leq ... \leq o^N$.
- Let k be the highest number such that $b^k \geq o^k$. Let

$$Z = \min\{b^k, o^{k+1}\} \tag{2}$$

and

$$z = \max\{o^k, b^{k+1}\} \tag{3}$$

Then

$$P(b^1, ..., b^N, o^1, ..., o^N) = (Z + z)/2$$

is the clearing price.

Payoffs

- Every buyer whose bid is at or above that price receives a unit at that price and every seller whose offer is at or below that price sells a unit at that price
- If $b^i \geq P$, (i.e., $i \geq k$), then i trades (i.e., is given a unit of the good) and receives a payoff of $V^i - P$.
- If $o^i \leq P$, (i.e. $i \geq k$), then i trades (i.e., gives up a unit of the good) and receives a payoff of $P - C^i$.
- All others do not trade and receive a payoff of 0.

Information Design

- Two market designs will be tested which differ only in their feedback to the agents.
- In the *Open Book Design*, each agent is given full information about all bids, offers and prices from the previous round: at the start of period t+1, each agent knows m_t and $P(m_t)$. So $s^i_{t+1} = [m_t, P(m_t)]$.
- In the *Closed Book Design*, agents are informed only about the price $P(m_t)$ in the previous round. So $s^i_{t+1} = P(m_t)$.

The Call Market in the Testbed

- How the foregone utilities are computed for a given $e = (V_1, ..., V_N, C_1, ..., C_N)$
- The information agents have is different in the two designs, what they can compute is also different
- Naive approach - implicitly assumed that agents treat the signals they see as if those signals will be repeated exactly in the next period
- There are no sophisticated expectations models or collection of historical signals.
- all we assume is "best" responses.

Closed book design

- each trader knows only the price from the previous period
- $s_t^i = P(m_t)$
- traders - price takers
- Given $P_t = P(m_t)$, each buyer can compute for each bid $a_{jt}^i \in A_t^i$

$$U(a_{j,t}^i, s_t^i) = \left\{ \begin{array}{l} V - P_t \\ 0 \end{array} \right\} if \left\{ \begin{array}{l} a_{jt}^i \geq P_t \\ a_{jt}^i < P_t \end{array} \right\}.$$

Given $P_t = P(m_t)$, each seller can compute for each offer $a_{jt}^i \in A_t^i$

$$U(a_{j,t}^i, s_t^i) = \left\{ \begin{array}{l} P_t - C^i \\ 0 \end{array} \right\} if \left\{ \begin{array}{l} a_{jt}^i \leq P_t \\ a_{jt}^i > P_t \end{array} \right\}.$$

Open book

- Each trader knows all the bids and offers from the previous period
- That is, $s_t^i = m_t$. Thus they can compute $W(m_t/a_{jt}^i)$ for each bid (offer) $a_{jt}^i \in A_t^i$.
- they can compute $U^i(a_j^i|s_t^i) = [V^i - P(m_t/a_j^i)]h_i(m_t/a_j^i)$ for a buyer:
  - take the bids and offers in m_t and rank and renumber them so that $b^{t,1} > b^{t,2} > ... > b^{t,N-1}$ and $o^{t,1} < o^{t,2} < ... < o^{t,N}$
  - Let k be the maximum number such that $b^{t,k} \geq o^{t,k}$.
  - need to compute what price would occur and whether we would trade, if we added a bid of b to these lists

Hypothetical utility of a bid b

If $b^{t,k} > o^{t,k+1}$

$$U(b|\cdot) = \left\{ \begin{array}{l} 0 \\ V - \frac{b+o^{t,k+1}}{2} \\ V - \frac{b^{t,k}+o^{t,k+1}}{2} \end{array} \right\} if \left\{ \begin{array}{l} b \leq o^{t,k+1} \\ o^{t,k+1} \leq b \leq b^{t,k} \\ b^{t,k} \leq b \end{array} \right\}$$

and if $b^{t,k} < o^{t,k+1}$

$$U(b|\cdot) = \left\{ \begin{array}{l} 0 \\ V - \frac{b+o^{t,k+1}}{2} \\ V - \frac{b^{t,k-1}+o^{t,k+1}}{2} \end{array} \right\} if \left\{ \begin{array}{l} b \leq b^{t,k} \\ b^{t,k} \leq b \leq b^{t,k-1} \\ b^{t,k-1} \leq b \end{array} \right\}.$$

- traders modeled as if they care about manipulating the equilibrium prices on the margin - they take into account the fact that their bids or offers may cause prices to change
- in contrast, in closed book design - traders modeled as price takers

Parameters

- $N = 5$ buyers and $N = 5$ sellers for each simulation
- The four sets of values, e, that provide a reasonable variation in the size of the margins, $Z - z$, and in the potential gains from trade:
- Each trader's mixed strategy set had $J = 100$ messages.
- The probability of experimentation was set to 0.033.
- for experimentation - drew from the normal distribution with the mean value equal to the value of the previous bid (offer) and standard deviation equal to 1.

Performance measures

- Efficiency
- Trading prices
- Values of individual bids and offers over the course of our simulations
- Efficiency is the ratio of the gains from trade in a call to the maximum possible gains from trade

$$E_t = \frac{\sum_{i=1}^{N} V_t^i h^i(m_t) - \sum_{j=1}^{N} C_t^j f^i(m_t)}{\sum_{i=1}^{N} V_t^i h^i(V, C) - \sum_{j=1}^{N} C_t^j f^i(V, C)}$$

IEL Results

- 4 different parameter sets, open and closed book
- efficiencies over 100 iterations, each lasted 20 periods
- the average efficiency levels are higher for the closed than for the open book design
- averaging across all iterations and parameter values, the closed book design - an efficiency of 92%; the open book design exhibited an efficiency of 83%
- Across the four parameter sets, the difference in efficiency between the closed and open book design ranged from 0.05 to 0.15.

IEL sims - parameter values

parameter set	1	2	3	4
buyer 1	1.00	0.90	1.00	2.00
buyer 2	0.93	0.70	0.95	1.80
buyer 3	0.92	0.50	0.90	1.60
buyer 4	0.81	0.30	0.80	1.40
buyer 5	0.50	0.022	0.15	1.20
seller 1	0.66	0.12	0.0	0.00
seller 2	0.55	0.37	0.05	0.20
seller 3	0.39	0.49	0.10	0.40
seller 4	0.39	0.87	0.20	0.60
seller 5	0.30	0.90	0.85	0.80

Average Efficiency Across 4 Parameter Sets

Set	Mechanism	Mean	Variance
1	closed	0.9164	0.0400
	open	0.8116	0.0456
2	closed	0.9227	0.0328
	open	0.8537	0.0470
3	closed	0.9405	0.0273
	open	0.8980	0.0349
4	closed	0.9256	0.0313
	open	0.7740	0.0257

Experimental Design

- We set up the experiments to exactly match the environments and mechanisms used for the simulations.
- Each observation comes from a group of 10 subjects: 5 buyers and 5 sellers
- Each group of subjects participated in a sequence of 8 phases.
- Each phase is identified with a particular environment, e, and a particular market design
- At the start of a phase, each subject was given a value (for buyers) or a cost (for sellers) that was fixed throughout the phase.
- In each phase only one type of call market (open or closed book) was used. The market was repeated 20 times in each phase.

Experiment Parameter Sets

phase	p1	p2	1	2	3	4	5	6	7	8
buyers										
1	400	400	50	90	100	180	130	193	120	180
2	400	400	92	50	80	190	170	150	180	160
3	400	400	93	70	15	195	190	181	160	140‘
4	400	400	100	30	95	115	159	200	140	200
5	400	400	81	10	90	200	110	192	200	120
sellers										
6	300	300	66	5	10	100	185	139	60	40
7	300	300	30	65	20	110	145	166	0	60
8	300	300	39	25	85	120	105	130	80	20
9	300	300	55	85	0	105	125	139	20	80
10	300	300	39	45	5	185	165	155	40	0

Experiment Parameter Values

phase	p1	p2	1	2	3	4	5	6	7
Type	C	O	C	O	O	C	C	O	C
Seconds/round	25	25	25	25	25	25	25	25	25
interim delay	5	5	5	5	5	5	5	5	5
startup delay	120	120	120	120	120	120	120	120	120
# rounds	10	10	20	20	20	20	20	20	20

The exchange rate was $0.006/franc

The total time of one session is 108 minutes.

The average payout/subject is $30 plus a $5 showup fee.

- group together data from each of phase 1 and 6, phase 2 and 5, phase 3 and 4, and phase 7 and 8.
- In each pair, one of the phases uses a closed book design and the other uses an open book design
- This allows for an easy comparison of the behavior observed for the same parameter values, but for different call market designs

Efficiencies - Human Data

parameter set	phases	open	closed
1	1,6	.89, .79	.98, .98
2	2,5	.92, .86	1.0, .76
3	3,4	.95, .9	.9, .95
4	8,7	.97	.98

Comparison of Simulations and Experiments

- Efficiencies are significantly higher in the closed book designs than in the open book designs (sims and exps)
- a lot more variation of the efficiency levels in the first periods (rounds) than towards the end which indicates the effects of learning (sims and exps)
- Both sims and exps exhibit fairly stable prices
- Bidding behavior
  - In both sims and exps, there is a tendency in the open book design for bidders to try to be strategically aggressive by bidding near the clearing price to try to manipulate that price.
  - There is much less tendency to do this in the closed book design and bids, especially of those holding intra-marginal units, tend to be nearer the true values or costs of the bidder.

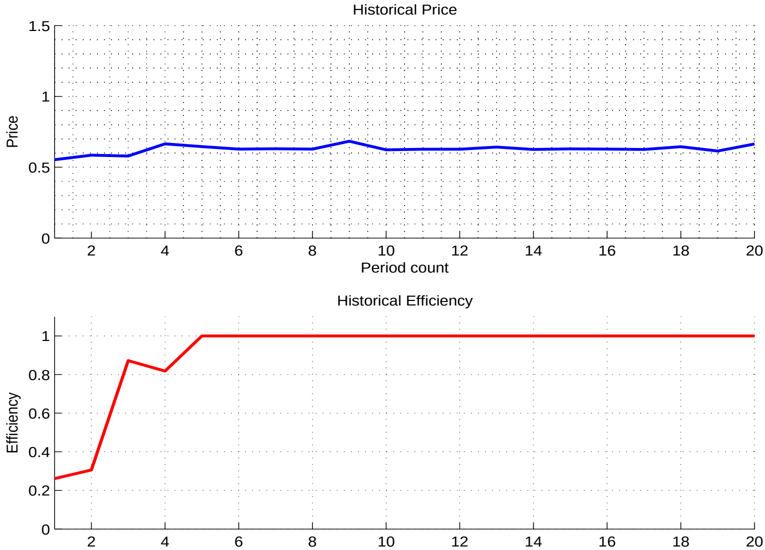
Comparison, cont.d

- Simulations are done in 'real time', i.e. the number of simulation periods is exactly equal to the number of experimental periods.
- Simulations initialized by choosing a random selection of initial strategies
- agents in IEL are very quickly producing patterns of bidding, prices and efficiencies similar to those from human experiments
- No other learning model able to do this.

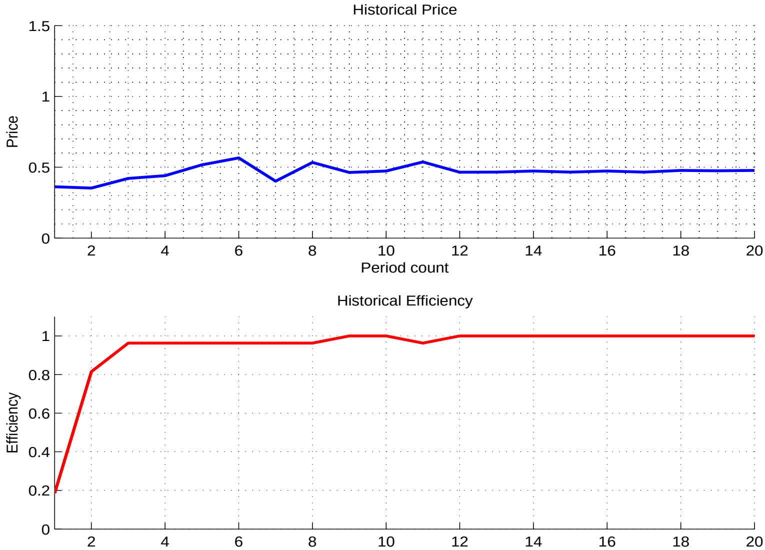
Comparison, cont.d

- A very simple market microstructure question: Is there difference in behavior exhibited in the closed and open book design?
  - Does the amount of information available to the traders affect the outcomes?
  - Our answer is yes.
- The closed book, which provides less information, leads to more efficient and less volatile outcomes. This answer is based on both simulations with our IEL testbed and experiments with humans subjects.

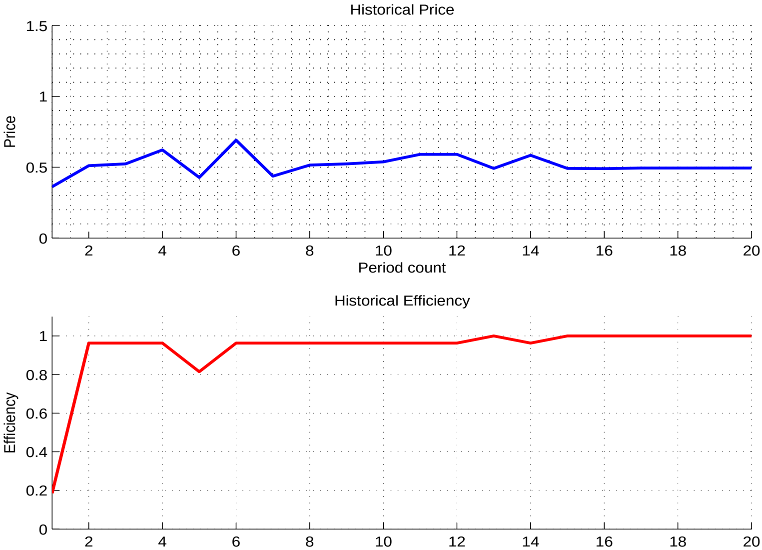
Price and Efficiency for Parameter Set 1 - Open Book



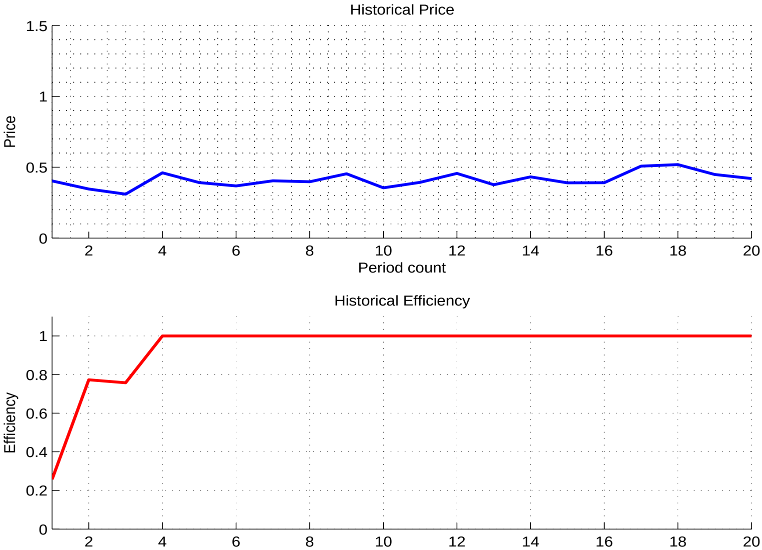
Price and Efficiency for Parameter Set 2 - Closed Book



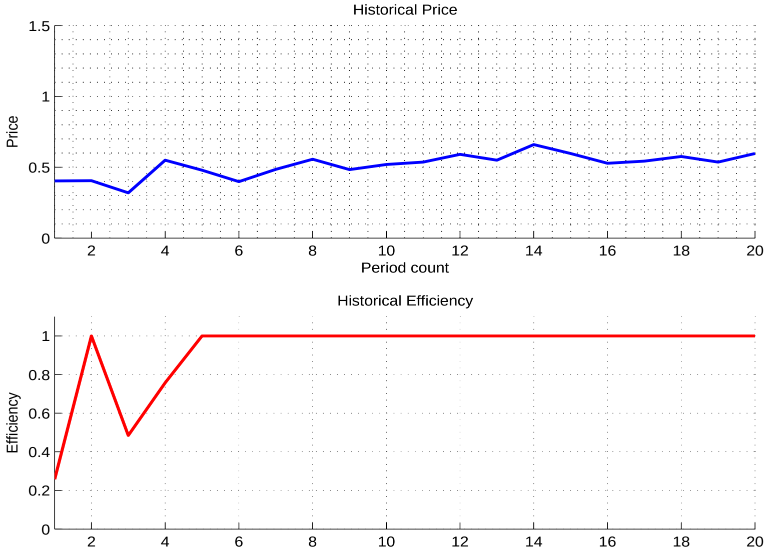
Price and Efficiency for Parameter Set 2 - Open Book



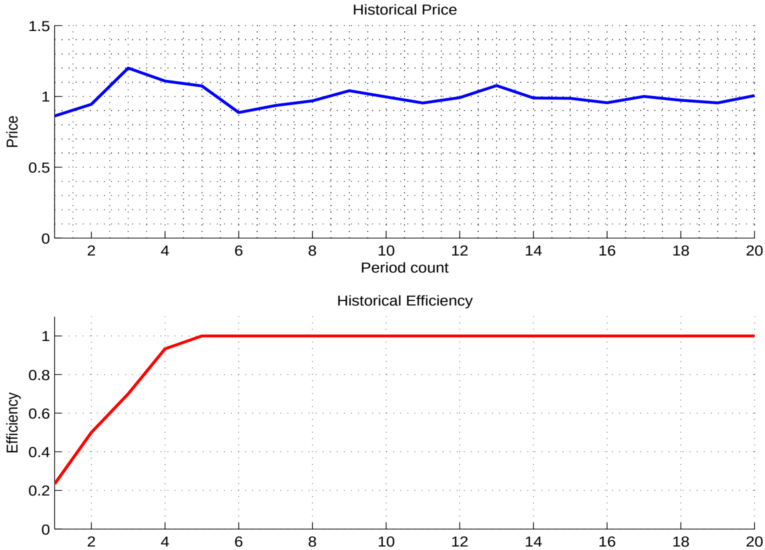
Price and Efficiency for Parameter Set 3 - Closed Book



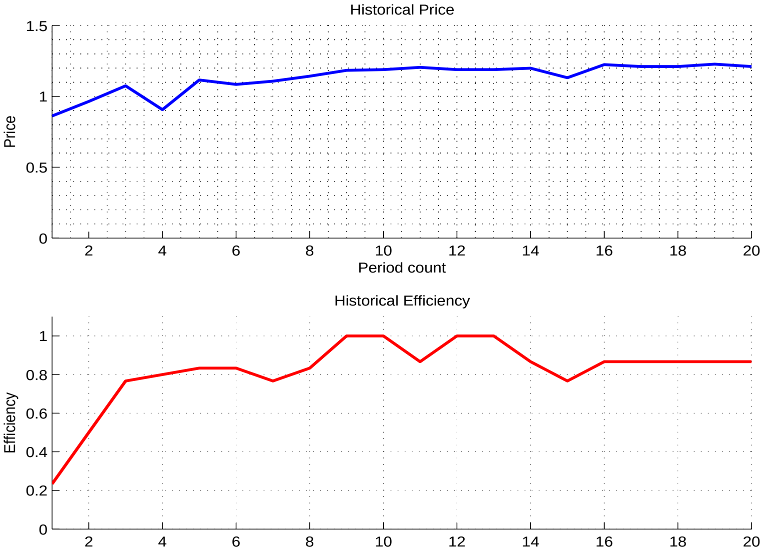
Price and Efficiency for Parameter Set 3 - Open Book



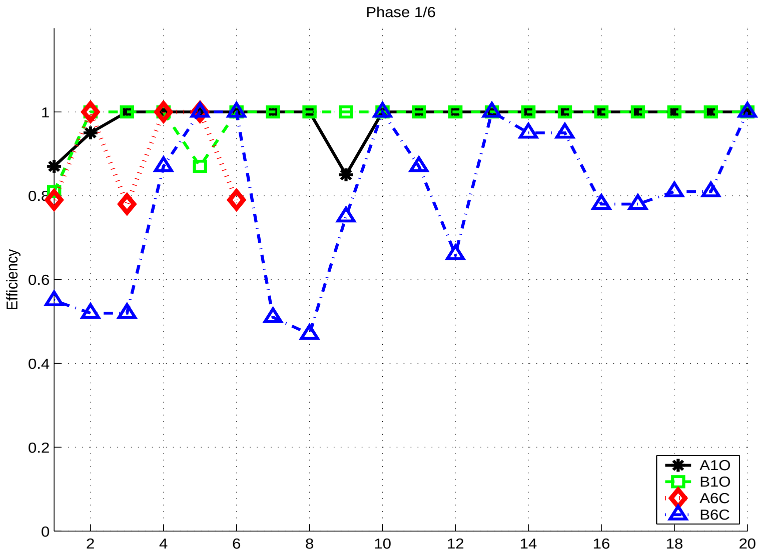
Price and Efficiency for Parameter Set 4 - Closed Book



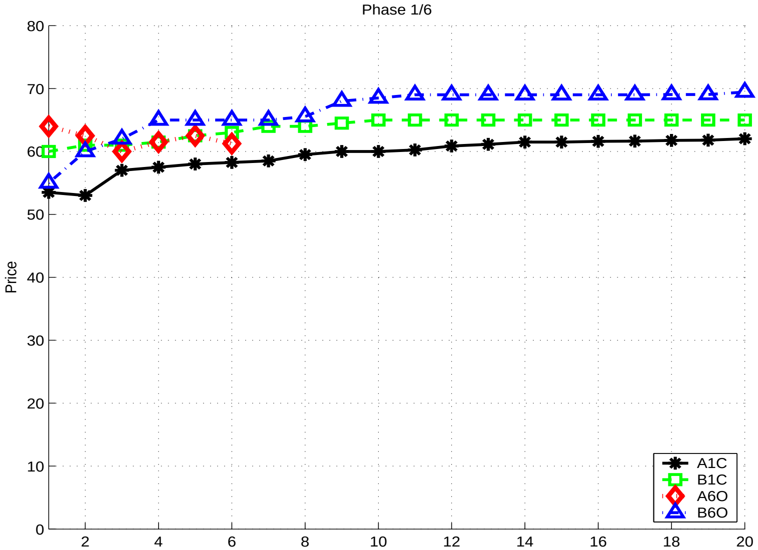
Price and Efficiency for Parameter Set 4 - Open Book



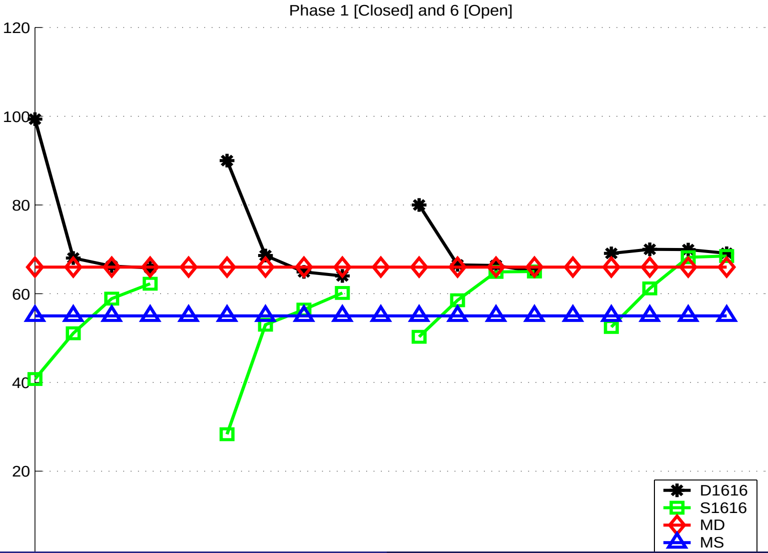
Efficiency for Phases 1 and 6



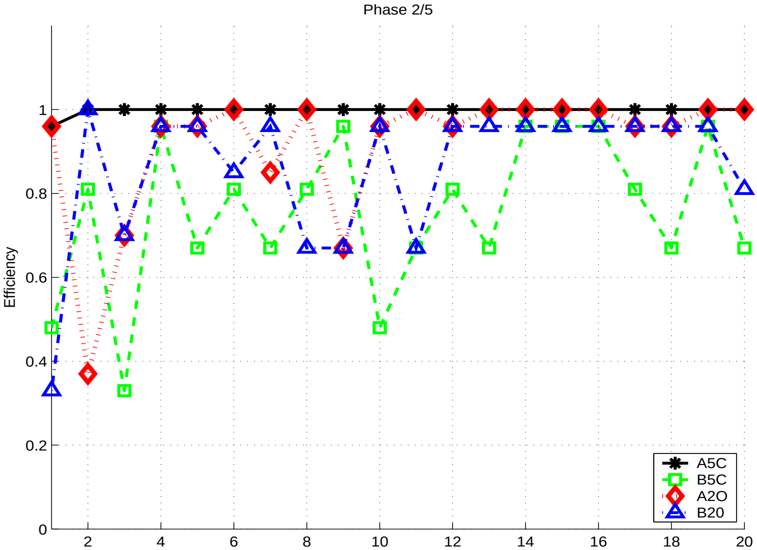
Prices for Phases 1 and 6



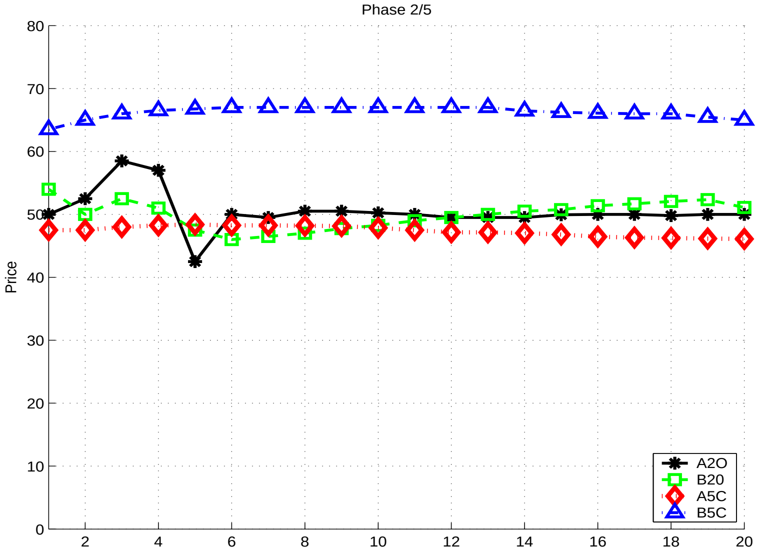
Submitted Bids and Offers - 1 and 6



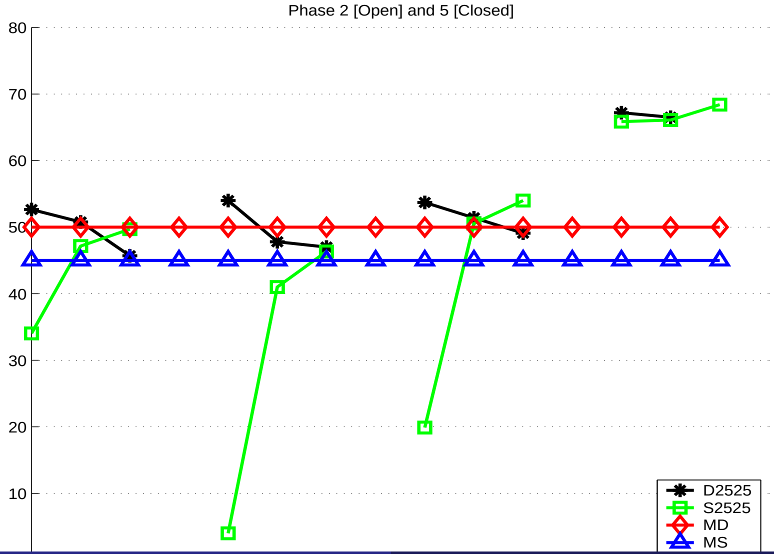
Efficiency for Phases 2 and 5



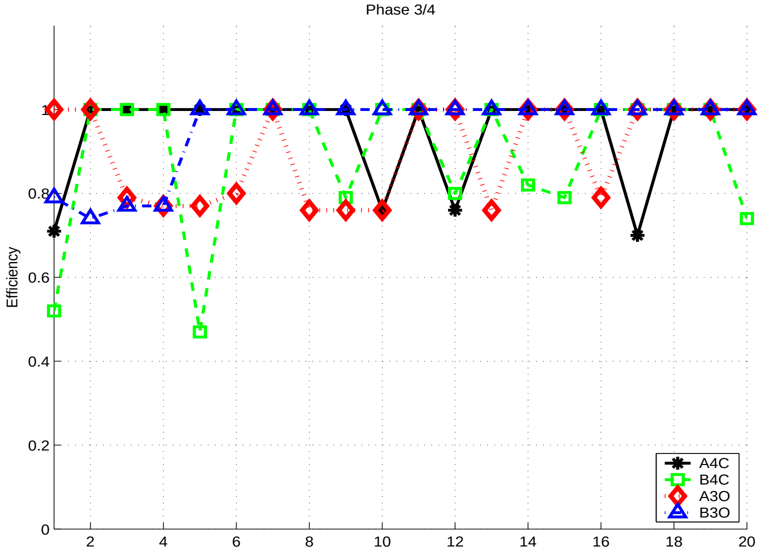
Prices for Phases 2 and 5



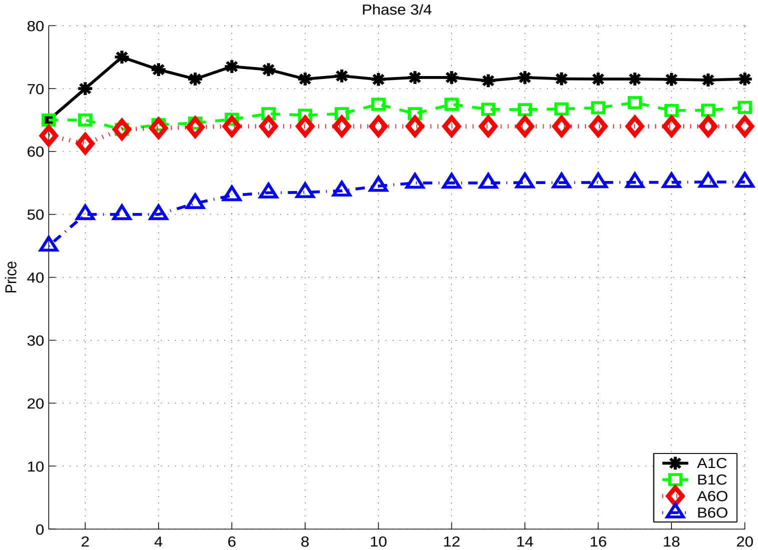
Submitted Bids and Offers - 2 and 5



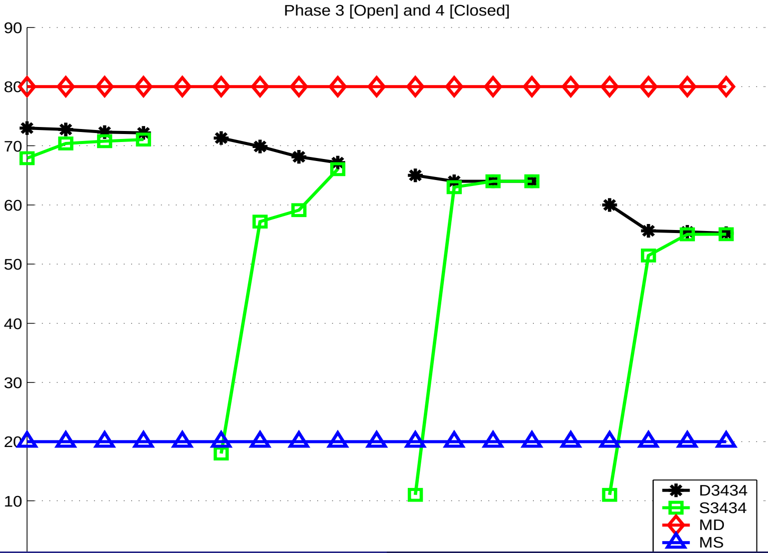
Efficiency for Phases 3 and 4



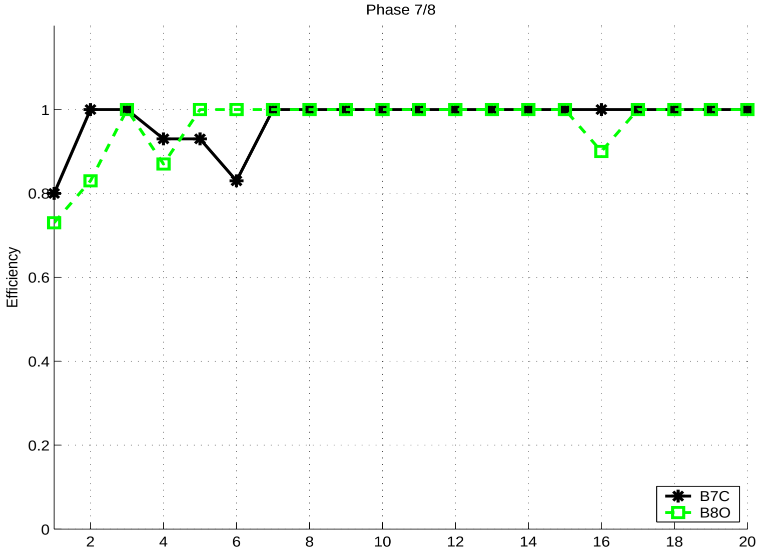
Prices for Phases 3 and 4



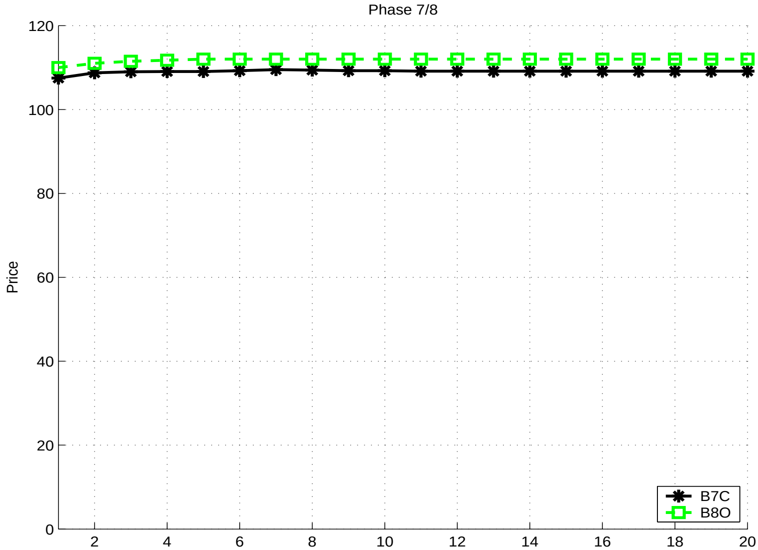
Submitted Bids and Offers - 3 and 4



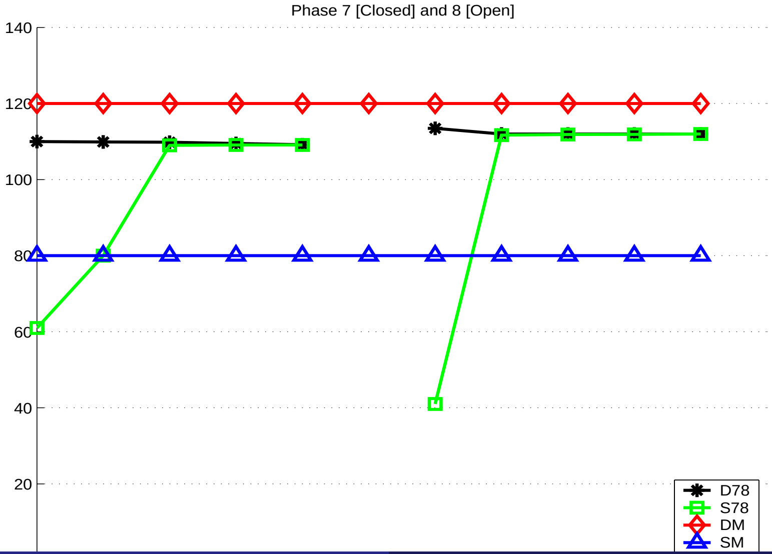
Efficiency for Phases 7 and 8



Prices for Phases 7 and 8



Submitted Bids and Offers- 7 and 8



One Market - Multiple Units

Homogenous Buyers and Sellers

Table 2

Simulation Results - Efficiency

Parameter Set	Auction Type	Mean	Variance
A.1	Closed	0.9164	0.0400
	Open	0.8116	0.0456
A.2	Closed	0.9227	0.0328
	Open	0.8537	0.0470
A.3	Closed	0.9405	0.0273
	Open	0.8980	0.0349

One Market - Multiple Units

Heterogenous Buyers and Sellers

Parameter Set	Auction Type	Mean	Variance
B.1	Closed	0.96055	0.02436
	Open	0.63118	0.03433
B.2	Closed	0.75889	0.03325
	Open	0.37286	0.05705
B.3	Closed	0.83421	0.02625
	Open	0.53346	0.00940
B.4	Closed	0.85447	0.02609
	Open	0.72634	0.01593

Multiple Markets, Single Unit

Heterogenous Buyers and Sellers

		Market				
Rules	Auction Type	1	2	3	4	5
100	Closed	0.9123 0.0316	0.9006 0.0393	0.9124 0.0268	0.9029 0.0375	0.9016 0.0364
100	Open	0.8590 0.0403	0.8607 0.0447	0.8682 0.0273	0.8559 0.0426	0.8631 0.0432
500	Closed	0.90778 0.03558	0.91327 0.03750	0.91136 0.03563	0.91574 0.03094	0.90574 0.03761
500	Open	0.8616 0.0381	0.8640 0.0392	0.8519 0.0398	0.8518 0.0363	0.8582 0.0417

Heterogenous Buyers and Sellers

Multiple Markets, Multiple Units

		Market				
Rules	Auction Type	1	2	3	4	5
100	Closed	0.7422	0.7522	0.7602	0.7659	0.7904
		0.0117	0.0111	0.0123	0.0108	0.0111
100	Open	0.5017	0.5139	0.5203	0.4796	0.4798
		0.0042	0.0039	0.0045	0.0033	0.0042
2500	Closed	0.8326	0.8703	0.8911	0.8788	0.8768
		0.0135	0.0303	0.0167	0.0169	0.0136
2500	Open	0.5597	0.5809	0.6004	0.5499	0.5948
		0.0082	0.0064	0.0071	0.0037	0.0058

Quadratic Utility

Multiple Units, Single Market

- Each agent is endowed a certain amount of each commodity, $e^i = (e^i_1, e^i_2, ..., e^i_k)$
- For each agent, $x^i_k > 0$ means the agent bought and $x^i_k < 0$ means they sold commodity k.
- The utility an agent gets from trading q^i at P is determined by:

$$W^i(x^i, P) = -(x^i + e^i)' H^i (x^i + e^i) + \mu^i (x^i + e^i) - P x^i + c^i$$

Quadratic Utility

- For each agent, a rule is now comprised of an integer representing the number of units they are bidding on (x^i) and a value representing the bid/ask over each the x^i units

Table 13
Quadratic Utility
Simulation Results - Efficiency

Parameter Set	Auction Type	Mean	Variance
Table 12	Closed	0.8976	0.0737
	Open	0.8419	0.0434