Complexity collaborators

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Coordinator of the General Integration Action in Complexity Science

GIACS | MORE IS DIFFERENT

General Integration of the Applications of Complexity in Science

Support for complexity students and activities etc.

like this ABM-S4-ESHIA thematic school
MORE IS DIFFERENT

General Integration of the Applications of Complexity in Science

(Anderson 72, Nobel 77) MORE IS DIFFERENT

Complex “Macroscopic” properties may be the collective effect of many simple “microscopic” components

E.G.

social and geographical wealth distribution, economic fluctuation (cycles, crashes, booms, stabilization by noise) may be explained and studied as the collective effect of many simple elementary transactions, investments, gains, losses between individual investors, traders, producers etc.
Phil Anderson  “Real world is controlled …
• by the exceptional, not the mean;
• by the catastrophe, not the steady drip;
• ….thus, we need to free ourselves from ‘average’ thinking.”

Simplest Example of a “More is Different” Transition

Water level vs. temperature

More is different: a single molecule does not boil at 100°C
Example of “MORE IS DIFFERENT” transition in Finance:

Instead of Water Level: -economic index (Dow-Jones etc…)

Crash = result of collective behavior of individual traders

Instead of temperature (energy / matter):
- Exchange rate/
- Interest rate
- Value at Risk /
- Liquid funds
- Equity Price /
- Dividends
- Equity Price /
- Fundamental value
- Taxation (without representation)/ Tea
**Cells**

- biology
- teleological causality
- life functions

**COMPLEXITY**

**EMERGENCE**

- reactions

**Molecules**

- chemistry
- past-to future causality

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**“MORE IS DIFFERENT”**

Complex Systems Paradigm

**MICRO** - the relevant elementary agents
- molecules

**INTER** - their basic, simple interactions
- reactions

**MACRO** - the emerging collective objects
- cells

**Intrinsically (3x) interdisciplinary:**

- **-MICRO** belongs to one science
- **-MACRO** to another science
- **-Mechanisms:** a third science

- chemistry
- biology
- statistical mechanics, physics
- math, game theory, info
Loss of Rationality at Crowd Level

Pareto economics Collective

- Zipf

Newton (after loosing 20 K Pounds in stock market)

“I can calculate the motions of heavenly bodies, but not the madness of people.”

Individuals individual behavior sell-buy (~rational (?) )

“MORE IS DIFFERENT”
Complex Systems Paradigm

MICRO - the relevant elementary agents traders
INTER - their basic, simple interactions orders, transactions
MACRO - the emerging collective objects herds, crashes, booms

Intrinsically (3x) interdisciplinary:
- MICRO belongs to one science Decision making, psychology
- MACRO to another science Financial economics
- Mechanisms: a third science statistical mechanics, physics math, game theory, info
Market makers

It was the kind of thing that sends shivers down economists' spines. On Friday, September 20, the stock market briefly went bananas. In 20 minutes, more shares were traded than are sometimes exchanged in a day. The FTSE 100 index, wobbling around 5,810 at 6:30am, hit a peak of 6,069 at around 10:15am, only to plummet to 5,735, minutes later. Fortunes were made and lost bycoffee

For traders, this was a strange event. But to a small group of physicists, the market “spike” was exactly what they’d expected. Not that anyone saw this particular event coming, but by applying their tools to economics, these “economophysicists” have demonstrated that huge fluctuations in the stock market are inevitable.

“The finance community

of it happening again.”

Other econophysicists believe they can explain why such giant wobbles happen. Economists look for a specific cause: one plausible culprit for the latest spout was a mistakenly huge trading order. But a single error cannot make the market index leap so high. This happens only when other traders jump into the fray. In other words, individual mistakes can trigger big fluctuations, but cannot create them. They depend on how those mistakes are related.

In an interact model, however, they do. They can act in particular, and becared for by what can happen. Such a model is the prescription for that occasionally market. Traders’ decisions, blind to each other, such inherently wobbling economies claim that the market might be chaos.

To physicists, huge market fluctuations are inevitable.

Lev Muchnik

is characteristic of a “highly damped oscillator”, a system that can oscillate but is muffled, like a piano string with the soft pedal.

Solomon’s team has been trying to understand the fluctuations of economic markets by using “interacting-agent” models developed from ideas in physics. In conventional

HARRY M. MARKOWITZ, Nobel Laureate in Economics

“If we restrict ourselves to models which can be solved analytically, we will be modeling for our mutual entertainment not to maximize explanatory or predictive power.”
Legends:

- Physicists come with preconceived models and force them on economic reality
- Physicists treat the models in a sloppy way with no regard to mathematical rigor or to analytic precise results.
- Physics models do not make contact to measurable economic quantities.

Answers

- Physicists come with preconceived models and force them on economic reality
- I will discuss a model introduced by economists 200 years ago and repeatedly and continuously used since.
- Physicists treat the models in a sloppy way with no regard to mathematical rigor or to analytic precise results.
- I will show that the model was mistreated until now and I will show how to treat it rigorously and extract analytically precise predictions that invalidate the previous beliefs.
- Physics models do not make contact to measurable economic quantities.
- I will compare the various predictions to empirical data and show perfect agreement.
**Malthus**: autocatalitic proliferation/ returns: \( B+A \rightarrow B+B+A \)

death/ consumption: \( B \rightarrow \emptyset \)

\[
\frac{db}{dt} = \alpha b
\]

\( \alpha = (\#A \times \text{birth rate} - \text{death rate}) \)

\( \alpha = (\#A \times \text{returns rate} - \text{consumption/losses rate}) \)

Exponential solution: \( b(t) = b(0)e^{\alpha t} \)

\( b = \frac{\#B}{\#A} \times \text{birth rate} > \text{death rate} \)

\( \alpha > 0 \)

\( \frac{db}{dt} = \alpha b - c b^2 \)

\( c = \text{competition/saturation} \)

Solution: exponential \( \Rightarrow \) saturation

**Verhulst** way out of it: \( B+B \rightarrow B \)

The LOGISTIC EQUATION
Diffusion of A at rate $D_a$
Diffusion of A at rate $D_a$
Diffusion of B at rate $D_b$
Diffusion of B at rate $D_b$

Diffusion of B at rate $D_b$
A+B → A+B+B; Birth of new B at rate $\lambda$

A+B → A+B+B; Birth of new B at rate $\lambda$
A+B \to A+B+B; \text{ Birth of new B at rate } \lambda

AB

A+B \to A+B+B; \text{ Birth of new B at rate } \lambda

AB
A+B \rightarrow A+B+B; Birth of new B at rate $\lambda$

A+B \rightarrow A+B+B; Birth of new B at rate $\lambda$

A\text{BB}

A\text{BBB}
A+B → A+B+B; Birth of new B at rate $\lambda$

ABB
BB

A+B → A+B+B; Birth of new B at rate $\lambda$

ABB
BB
A+B → A+B+B; Birth of new B at rate $\lambda$

Another Example

B

A A

A+B → A+B+B; Birth of new B at rate $\lambda$

Another Example

B

A A
A+B \rightarrow A+B+B; \text{ Birth of new } B \text{ at rate } \lambda

Another Example

B
AA

A+B \rightarrow A+B+B; \text{ Birth of new } B \text{ at rate } \lambda

Another Example

B
AA

20/09/2007
A+B → A+B+B; Birth of new B at rate $\lambda$

Another Example

BBB
AA

A+B → A+B+B; Birth of new B at rate $\lambda$

Another Example

BBB
AA
B → ∅  
Death of B at rate $\mu$

\[\begin{array}{c}
\text{B} \\
\end{array}\]
\[ B \rightarrow \emptyset \quad \text{Death of } B \text{ at rate } \mu \]

\[ B \rightarrow \emptyset \quad \text{Death of } B \text{ at rate } \mu \]
B+B $\rightarrow$ B; Competition of B’s at rate $\gamma$
B+B $\rightarrow$ B; Competition of B’s at rate $\gamma$
B+B \rightarrow B; Competition of B’s at rate $\gamma$

Interpretations in Various Fields:

Origins of Life:
- A = proteins
- B = DNA

Genetic Evolution:
- Sites: various genomic configurations.
- B = individuals; Jumps of B = mutations.
- A = advantaged niches (evolving fitness landscape).

Immune system:
- sites = shapes / strains of antigens and antibodies
- B = cells, antibodies;
- A = antigen
  B cells that meet corresponding antigen multiply.
almost all the social phenomena, except in their relatively brief abnormal times obey the logistic growth.

“Social dynamics and quantifying of social forces”
Elliott W. Montroll
US National Academy of Sciences and American Academy of Arts and Sciences

'I would urge that people be introduced to the logistic equation early in their education...
Not only in research but also in the everyday world of politics and economics …”
Nature
Robert McCredie, Lord May of Oxford, President of the Royal Society

Reality:
survival, time fluctuations

Spatial Patterns

Diff Eq prediction; spatial uniformity
Misfit was always assigned to the neglect of **specific details**. We show it was rather due to the neglect of the **discreteness**.

Once taken in account => complex adaptive collective objects.
emerge even in the worse conditions

\[
d X_i = (a_i + c_i(X_i,t)) X_i + \sum_j a_{ij} X_j
\]
\[ dX_i = (\text{rand}_i(t) + c_i(X_i, t)) X_i + \sum_j a_{ij} X_j \]

At each time instance

\[ P(X_i) \sim X_i^{-1-\alpha} \]

At short times

\[ P(\Delta X) \sim (\Delta X)^{-1-\alpha} \]

\[ P(\Delta X=0) \sim (\Delta t)^{-1/\alpha} \]

Fractal Fluctuations
\[ dX_i = \left( \text{rand}_i + c_i(X_i,t) \right) X_i + \sum_j a_{ij} X_j = \]
one can prove rigorously (by Renorm. group)
Shnerb, Louzoun, Bettelheim, Solomon (PNAS 2000)

Branching Random Walks Theorems (EJP2002)) that:
- On a large enough 2 dimensional surface,
  the B population *always grows!*

No matter how fast the death rate $\mu$,
how low the A density,
how small the proliferation rate $\lambda$
- In all dimensions $d$:
  \[ \frac{\lambda}{D_a} > 1 - P_d \]
  always suffices; $P_d = \text{Polya's constant}$

Discrete Individuals $\rightarrow$ **microscopic noise**

Autocatalytic proliferation $\rightarrow$ **amplification**

**Collective Macroscopic Objects**

- **Emergent Properties : Adaptability**
- **Life always wins in 2 D**

- Most singular, rarest fluctuations dominate
  the system dynamics

- **Power Laws: - wealth distribution**
- **Levy market fluctuations**
First round of intuitions on the effect

A one-dimensional simple example
continuum prediction

\[ a = \frac{2}{14} \quad \lambda = 1 \quad \mu = \frac{1}{2} \]
A one-dimensional simple example

continuum prediction

\[ a \times \lambda - \mu \]

\[ 2/14 \times I - 1/2 = -5/14 \]

A one-dimensional simple example

discrete prediction

\[ b_{4}(t+1) = (1 + 1 \times \lambda - \mu) \, b_{4}(t) \]

\[ b_{13}(t+1) = (1 + 0 \times \lambda - \mu) \, b_{13}(t) \]
A one-dimensional simple example

Discrete prediction

Initial exponential decay

Continuum prediction

\[
\begin{align*}
(\frac{3}{2}) & \times (\frac{3}{2}) \\
(\frac{1}{2}) & \times (\frac{1}{2})
\end{align*}
\]

\[
\begin{align*}
A & \rightarrow \begin{array}{c}
\text{B level}
\end{array}
\end{align*}
\]

\[
\begin{align*}
(\frac{3}{2}) & \times (\frac{3}{2}) \\
(\frac{1}{2}) & \times (\frac{1}{2})
\end{align*}
\]

\[
\begin{align*}
A & \rightarrow \begin{array}{c}
\text{B level}
\end{array}
\end{align*}
\]

\[
\begin{align*}
(\frac{3}{2})^2 & \times (\frac{3}{2})^2 \\
(\frac{1}{2})^2 & \times (\frac{1}{2})^2
\end{align*}
\]

\[
\begin{align*}
A & \rightarrow \begin{array}{c}
\text{B level}
\end{array}
\end{align*}
\]
A one-dimensional simple example

discrete prediction

\[(3/2)^t\] growth

continuum prediction

\[(9/14)^t\]

\[t\]

...
A one-dimensional simple example

The Role of DIFFUSION

The Emergence of Adaptive B islands

Take just one A in all the lattice:
B diffusion
Growth stops when A jumps to a neighboring site
WIDEN THE B ISLAND LINEARLY IN TIME

B population on the **old A** site will decrease

Growth will start on the **New A** site
Growth stops when $A$ jumps again (typically after each time interval $1/D_A$)
Growth stops again when A jumps again (typically after each time interval $1/D_A$)
If the time interval $d/D_A$ is small, when $A$ jumps back it still finds $B$’s to proliferate.

Emergent Collective Dynamics:
- **B-islands** search, follow, adapt to, and exploit fortuitous fluctuations in $A$ density.

This is in apparent contradiction to the “fundamental laws” where individual $B$ don’t follow anybody.

The strict adherence of the elementary particles $A$ and $B$ to the **basic fundamental laws** and the **emergence** of complex adaptive entities with **self-serving behavior** do not interfere one with another. Yet they determine one another.

**Is this a mystery?** Not in the AB model where all is on the table!
Interpretations in Many Other Fields and Systems

Genetic Evolution:
- Sites: various genomic configurations.
- B= individuals; Jumps of B= mutations.
- A= advantaged niches
- emergent adaptive patches= species

Origins of Life:
- individuals = chemical molecules,
- adaptive patches = first self-sustaining proto-cells.
  even with very low protein density and
  very low reproduction RNA capability and
  very unstable RNA structure,
self–copying RNA would have survived if earth was large enough!

Cells, species, companies, are “tricks” of life / economy
to survive in conditions that are in average in-survivable.
Newton (after loosing 20 K Pounds in stock market)
“I can calculate the motions of heavenly bodies, but not the madness of people."

But from the above analysis:
Financial markets don't need wise/ intelligent investors to work:
   Capital can survive and even proliferate simply by being autocatalytic
Adam Smith’s invisible hand… doesn’t even need investor’s self-interest

Indeed, according to the above effects:
One has automatic (not even deliberate) Selection of the medium by the animals (and of the profitable investment by investors):
Even though most of the medium is inhabitable (most investments are not profitable), since the parents (capital) survive only in habitable (profitable) places (investments) the new generation (reinvestment of capital) is born (re-invested) only at the “right” / survivable/ profitable locations!
Other applications

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>Clustered Adoption: Prediction Tool for marketing success (15/17)</th>
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<tbody>
<tr>
<td>At the transition, the sales are fractal: = non uniform space-time distribution.</td>
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CARS in USA
Jacob Goldenberg
- Microscopic Immune Cells and Macroscopic Health
MICRO - Cells, Enzymes, Antigens, Antibodies
INTER - producing, destroying, changing state of cells/enzymes
MACRO - immunity, health, infection, sickness, inflammation.

Losing All Battles and Wining the War; HIV time hierarchy:
U Hershberg, Y Louzoun, H Atlan and S Solomon

Antigen-receptor degeneracy and immunological paradigms
Irun R. Cohen, Uri Hershberg, Sorin Solomon
Molecular Immunology 40 (2004) 993–996

- Microscopic seeds and Macroscopic Oases
MICRO – individual plants
INTER – growth, water fixation,
MACRO – bushes, vegetation patches

N.M. Shnerb, P. Sarah, H. Lavee, and S. Solomon
Small changes in product quality, price, external conditions can produce large effects (e.g. large market fluctuations)

Small deterioration in credit market can trigger large waves of bankruptcies

Wealth distribution scaling Vs Market index returns scaling

Stock market shock explained. Physicists model recent trading frenzy.
Multi-Agent stochastic \( \langle \alpha \rangle \ll 0 \) prediction 

Logistic Equations

GDP Poland

Multi-Agent Prediction Confirmed Experimentally

Nowak, Rakoci, Solomon, Ya’ari

GDP per capita in 1990 units (of each country)

Year
Number of Economic Enterprises per capita 1989

B = Number of Economic Enterprises per capita 1994

“A” = education 1988

Nowak, Rakoci, Solomon, Ya’ari
“A” = education

Number of Economic Enterprises per capita

1988

1989
Conclusions

• The logistic dynamics was believed for 200 years to be capable to describe a very wide range of systems in biology, society, economics, etc

• The naïve continuous differential equations expression of this dynamics lead often to predictions incompatible with the empirical evidence

• We show that taking properly into account the multi-agent character of the system one predicts generically the emergence of adaptive, collective objects supporting development and sustainability.

• The theoretical predictions are validated by the confrontation with the empirical evidence and are relevant for real life economic, social and biological applications.
50 min up to here in real lecture
First year of liberalization

Enterprize per capita correlation to…
Growth rate correlation to…

Enterprize per capita spatial correlation
Growth rate spatial correlation

Second-Third year of liberalization

Enterprize per capita correlation to…
Growth rate correlation to…

Enterprize per capita spatial correlation
Growth rate spatial correlation
Forth and further years

Enterprise per capita correlation to...
Growth rate correlation to...

Enterprise per capita spatial correlation
Growth rate spatial correlation

per capita GDP v. Avq IQ
linear regression

per capita GDP v. Avq IQ
number of enterprises per capita in groups of counties
**Other predictions**

- **Case 1:** low level of capital redistribution  
  - high income inequality  
  - outbreaks of instability (e.g. Russia, Ukraine).

- **Case 2:** high level of central capital redistribution  
  - slow growth or even regressing economy (**Latvia**) but quite  
  - uniform wealth in space and time.

- **Case 3:** **Poland** - optimal balance:  
  - transfers enough to insure adaptability and sustainability  
  - yet the local reinvestment is enough to insure growth.

![Graph showing economic trends in Poland, Russia, Ukraine, and Latvia](image)
Spatial fluctuations of $b$ between different sites
⇔
Time fluctuations in the total $b$
Spatial fluctuations of \( b \) between different sites 
\( \Leftrightarrow \) Time fluctuations in the total \( b \) 
Exponent of distribution of individual wealth = (Prediction) 
Exponent of distribution of market fluctuations

Salaries/(speculative fluctuations); dependents/salary; minimal income /average income; pop. increase/ gener
What next?

Measure chain of changes in capital growth and transfer due to Fiat plant closure.

Enterprises creation and disappearance, etc.

Check alternatives.
Economic development

Piemonte

Measure chain of changes in capital growth and transfer due to Fiat plant closure.
Enterprises creation and disappearance, etc.

Check alternatives

EXTENSIONS OF THE AB – Like MODELS

The ALPHABET Model (the AAB Model)

In general, the conditions necessary are of many types, with different time scales, death rates, diffusion rates and densities.

The B’s can develop only if all these growth factors coincide in space at a given time: very rare events but very important.

The naive differential equation would be:

\[ \dot{b} = (\prod a_i \lambda - \mu) b + D_b \Delta b - \chi b^2 \]

but as before the dynamics will be dominated by the rarest fluctuations in the \( a_i \)'s configuration.
\( \text{A1+A2+A3+B} \rightarrow \text{A1+A2+A3+B+B} \)

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\( \text{A1+A2+A3+B} \rightarrow \text{A1+A2+A3+B+B} \)

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OTHER EXTENSIONS OF THE AB – Like MODELS

The ABB Model

This model tries to explain the process of discontinuous discovery of new technologies by considering nonlinear dependence of the B production rate on the B’s:

\[
\frac{db}{dt} = \text{Polynomial of } b = a_0 - a_1 b + a_2 b^2 + a_3 b^3 + \ldots
\]

If the system starts at the smallest zero, a rare fluctuation can promote it irreversibly to the next (stable) zero and so on.
OTHER EXTENSIONS OF THE AB – Like MODELS

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\[ \text{Db} / \text{dt} = \text{Polynomial of b} \]

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OTHER EXTENSIONS OF THE AB – Like MODELS

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\[ \frac{d b}{dt} = \text{Polynomial of } b \]

If the system starts at the smallest zero, a rare fluctuation can promote it irreversibly to the next (stable) zero and so on.

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Losing All Battles and Wining the War

**HIV time hierarchy:**

U Hershberg, Y Louzoun, H Atlan and S Solomon


A = antigens (virons)

B = cells of the immune system

i = index of the particular characteristic shape of virus/immune cell

\[ A_i \rightarrow A_i + A_i \]  Virons multiply

\[ A_i + B_i \rightarrow A_i + B_i + B_i \]  Immune cells multiply when they meet virons with complementary shape to theirs

\[ B_i + A_i \rightarrow B_i \]  Virons are destroyed when detected by immune cells of complementary shape

\[ A_i \rightarrow A_{i\pm1} \]  Virons can mutate (actually in a n-dim space)

\[ A_i + B_s \rightarrow A_i \]  Immune cells of any type are destroyed when infected by viruses of any type

The immune system generates cells with various characteristic shapes to probe for the presence of antigens with complementary shapes.
Once some **virons** get in the system, they **multiply** unhindered as long as none of them meets an immune system cell with complementary shape.

Once one viron (individual from the strain) meets an immune system cell **the cell keeps multiplying** and its descendents meet more virons and multiply too.

Some **mutant virons** with different shape (and therefore undetectable by the present strain of immune cells) are **produced**.
The virons from the strain detected by the cells with complementary shape are destroyed.

The mutant ones have different shape. They are not detected (yet) so they multiply unhindered.

The detected viron strain is destroyed by the immune system.

Shape space
The detected viron strain is destroyed by the immune system.

Shape space

The detected viron strain is destroyed by the immune system.

Shape space
Before being completely destroyed, the detected strain is able to generate randomly more mutants, with different characteristic shape.

The initial strain is decimated but the mutants are still undetected and multiply unhindered.
The initial strain has now disappeared. The acute phase: primary infection, is finished. The mutants are still undetected.

This strain has so small population that even an immune cell with complementary shape doesn’t meet/detect any of its individuals.

After the initial strain is destroyed, the immune cells with complementary shape do not meet any excitation and they die without multiplying. Some “memory cells” with the information of the initial strain shape are left (forever).

In the meantime one of the mutant strains is detected
The immune cells with the complementary shape to the detected strain multiply. They are not many enough yet to stop the multiplication of the strain and in particular the generation of some mutants.
The detected strain is being decimated but its mutants do well and in fact produce mutants of themselves.

The detected strain is about to disappear and another strain is just being detected.
The antibodies corresponding to the destroyed strain disappear. Only memory cells are left.

Antibodies corresponding to the newly detected strain are being produced.
The virus loses another battle but the number of strains keeps increasing. *Copy at the beginning*
The virus looses another battle but the number of strains keeps increasing until it overcomes the immune system.
Each point represents another AB system: the coordinates represent its parameters: naïve effective B decay rate \((\mu - \lambda a_0)\) and B division rate \(\lambda\).

**Positive Naïve Effective B decay rate**
\[
= (\mu - \lambda a_0)
\]

**Death**

**Life;**

**Negative Decay Rate**
\[
= \text{Growth}
\]

Initially, at small scales, B effective decay rate increases.

At larger scales B effective decay rate decreases.

Life wins!
Pólya ‘s Random Walk Constant

What is the probability $P_d(\infty)$ that eventually an A returns to its site of origin?

Pólya: $P_1(\infty) = P_2(\infty) = 1$

but for $d>2$ $P_d(\infty) < 1$; $P_3(\infty) = 0.3405373$

Kesten and Sidoravicius studied the AB model (preprint 75 p):

On large enough 2 dimensional surfaces $\forall \lambda, \mu, D_b, D_a, a_0$

Total/average B population always grows.
Study the effect of one A on $b(x,t)$ on its site of origin $x$

Ignore for the moment the death and emigration and other A’s

Probability of one A return by time $t$ (d-dimesional grid): $P_d(t)$

Typical duration of an A visit: $1/D_a$

Average increase of $b(x,t)$ per A visit: $e^{\lambda/D_a}$

Expected increase in $b(x,t)$ due to 1 return events: $e^{\lambda/D_a} P_d(t)$

But for $d=2$ $P_d(\infty) = 1$

so $e^{\lambda/D_a} P_d(\infty) > 1$

$\eta \sim \lambda/2D_a > 0$

$\tau \sim e^{D_a/2\lambda} < \infty$

finite positive expected growth in finite time!
\{\text{Probability of } n \text{ returns before time } t = n \tau\} > P^n_d(\tau)

Growth induced by such an event: \(e^{n\lambda/D_a}\)

Expected factor to \(b(x,t)\) due to \(n\) return events:

\[> \left[ e^{\lambda/D_a} P_d(\tau) \right]^n = e^{n \eta} = e^{\eta t/\tau}\] exponential time growth!

Taking in account death rate \(\mu\),

- emigration rate \(D_b\) and that

\(< b(x,t) > \gg b(x,0) e^{-(\mu + D_b) t} e^{a(x,0) \eta t/\tau}\)

-increase is expected at all \(x\)'s where: \(a(x,0) > (\mu + D_b) \tau/\eta\)

There is a finite density of such \(a(x,0)\)'s \(\Rightarrow <b(x,t)> \rightarrow \infty\)

\[\text{EXTENSIONS OF THE AB – Like MODELS-}\]

Rare but irreversible and unavoidable events

The ABB Model

This model tries to explain the process of discontinuous discovery of new technologies by considering nonlinear dependence of the B production rate on the B’s:

\[Db/\text{dt} = \text{Polynomial of } b = a_0 - a_1 b + a_2 b^2 + a_3 b^3 + \ldots\]

If the system starts at the smallest zero, a rare fluctuation can promote it irreversibly to the next (stable) zero and so on
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